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# Dynamic Complex-Frequency Control of Grid-Forming Converters

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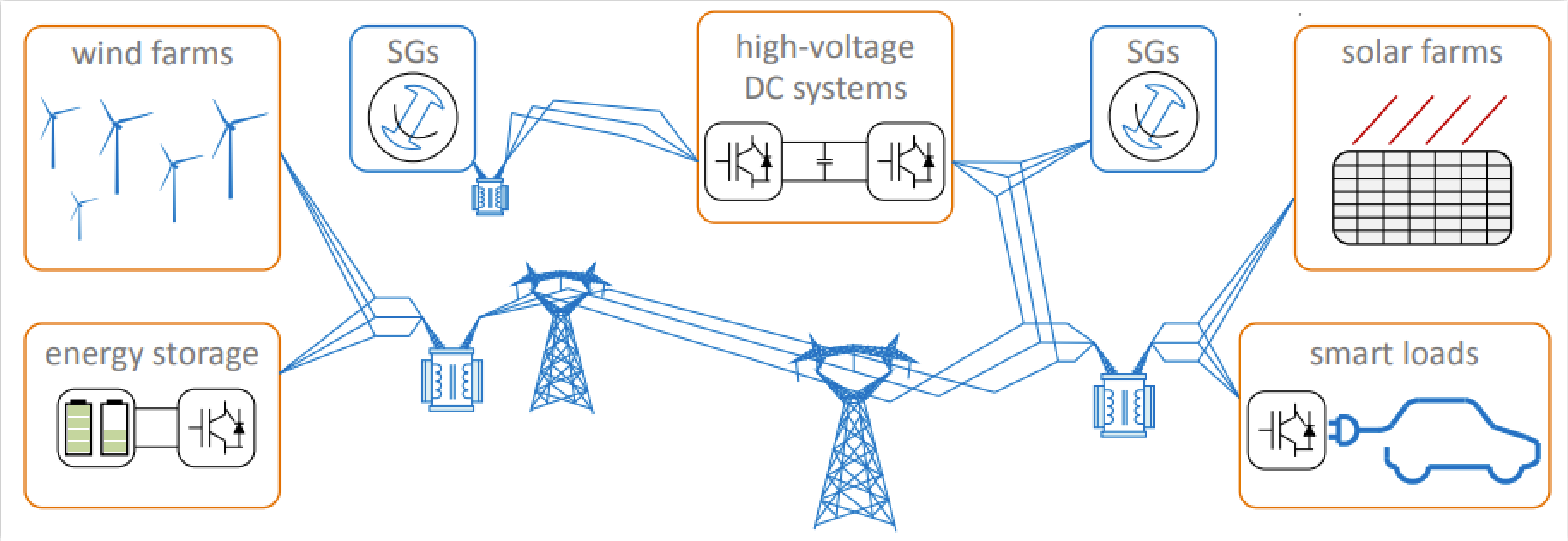
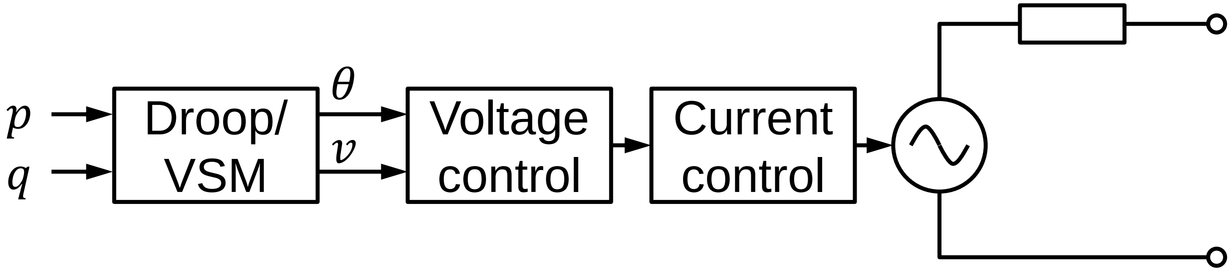
# Grid-forming inverters are crucial!



# Classical grid-forming is not perfect…

- Droop control and VSM

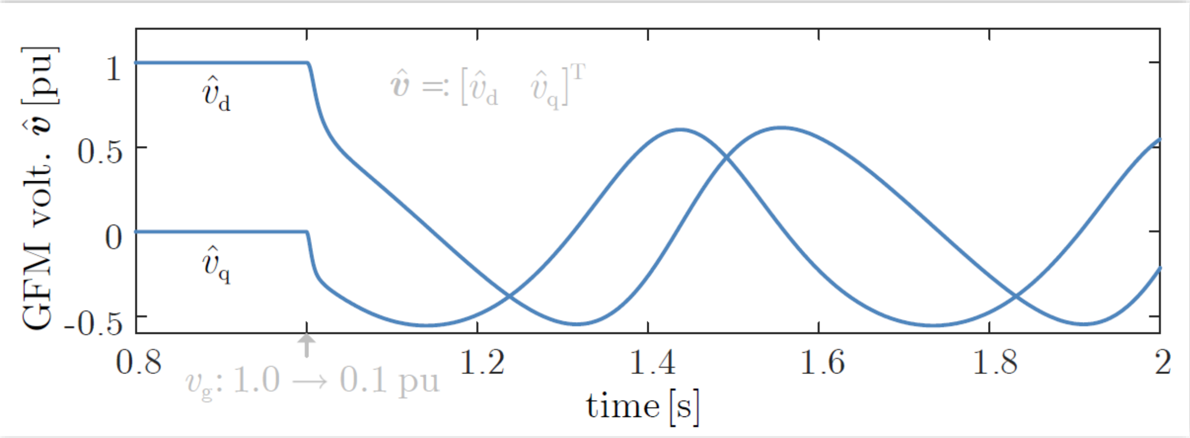
$$\dot{\theta} - \omega_0 = \eta(p^* - p)$$
$$\text{or } T_J\ddot{\theta} + D(\dot{\theta} - \omega_0) = p^* - p$$
$$\dot{v} = \eta\alpha(v^* - v) + \eta(q^* - q)$$




- Single-input single-output: based on **decoupling** and **linearization**
- Good power-sharing property and small-signal performance
- **Poor large-signal performance**

→**Need multivariate/MIMO controllers**

$v_g = 0.1, z_g = 0.4 + j0.4, p^* = 0, q^* = 0, v^* = 1, \alpha = 1, \eta = 0.08$



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2. Dynamic Complex-Frequency Control
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# Complex-frequency control

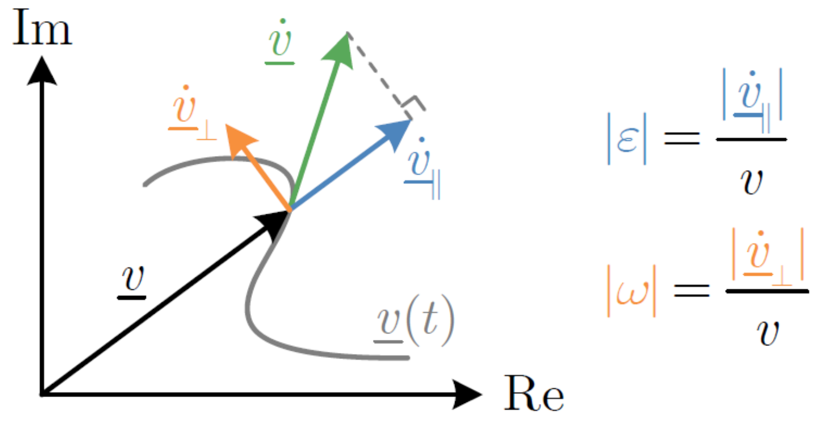
- **Complex variables are multivariate**
  - Complex voltage $\underline{v} = v e^{j\theta}$
  - Complex angle $\underline{\vartheta} := \ln v + j\theta$
  - Complex power $\underline{s} = p + jq$
  - Complex frequency $\underline{\varpi} := \dot{\underline{\vartheta}} = \frac{\dot{v}}{v} + j\dot{\theta}$

- **Complex droop control are multivariate**
  - Multivariate feedback: complex power
  - Multivariate control output: complex angle

$$\dot{\underline{\vartheta}} = j\omega_0 + \eta e^{j\varphi}\left(\frac{p^\star - jq^\star}{v^{\star 2}} - \frac{p - jq}{v^2}\right) + \eta\alpha\frac{v^{\star 2} - v^2}{v^{\star 2}}$$

VS.

SISO droop control: $\dot{\theta} = \omega_0 + \eta\left(p_\varphi^\star - p_\varphi\right)$




Complex frequency $:= \varepsilon + j\omega$

# Comparison with classics

| | Classical droop control | Complex droop control |
|---|---|---|
| Controller | $\dot{v} = \eta\,(q^\star - q) + \eta\alpha\,(v^\star - v)\,,$<br>$\dot{\theta} = \omega_0 + \eta\,(p^\star - p)\,.$ | $\frac{\dot{v}}{v} = \eta\left(\frac{q^\star}{v^{\star 2}} - \frac{q}{v^2}\right) + \eta\alpha\frac{v^\star - v}{v^\star}, \text{or} + \eta\alpha\frac{v^{\star 2} - v^2}{v^{\star 2}},$<br>$\dot{\theta} = \omega_0 + \eta\left(\frac{p^\star}{v^{\star 2}} - \frac{p}{v^2}\right).$ |
| Stability | Small-signal stability is guaranteed but transient stability not guaranteed (far away from the nominal) | Both small-signal stability and transient stability are theoretically guaranteed |
| Dynamic performance | Dynamic droop control: VSM and their variants | Dynamic complex-frequency droop control |
| Transient performance | Virtual admittance + limiter<br>Cross-forming control<br>Threshold virtual impedance | Virtual admittance + limiter<br>Cross-forming control<br>Threshold virtual impedance control |
| Steady-state performance | Power sharing | Power sharing |

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# Dynamic complex-frequency control

- **Static droop**
  - Static gains
  - No inertia response

$$\underline{\dot{\vartheta}} = j\omega_0 + \eta e^{j\varphi}\left(\frac{p^\star - jq^\star}{v^{\star 2}} - \frac{p - jq}{v^2}\right) + \eta\alpha\frac{v^{\star 2} - v^2}{v^{\star 2}}$$

$$\begin{bmatrix}\Delta\varepsilon\\ \Delta\omega\end{bmatrix} = \begin{bmatrix}0 & -\eta\\ \eta & 0\end{bmatrix}\left(\begin{bmatrix}-\Delta\rho\\ \Delta\sigma\end{bmatrix} - \begin{bmatrix}0\\ -\alpha\end{bmatrix}\frac{\Delta v}{v^*}\right)$$

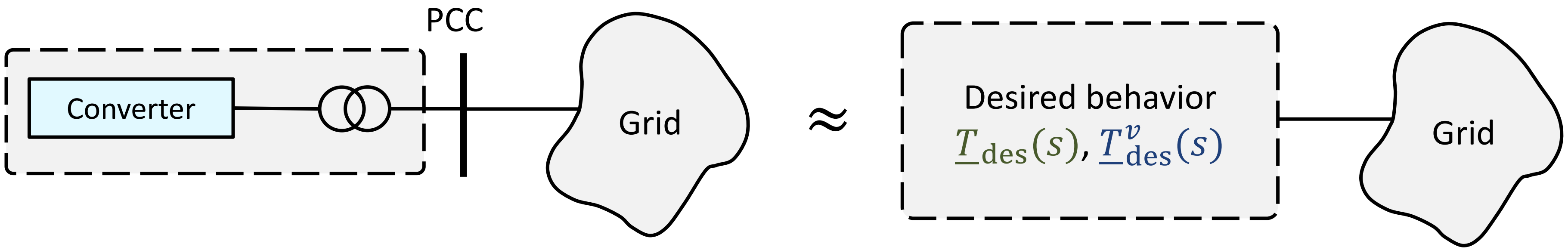
vs.

- **Dynamic droop**
  - Dynamic gains (Transfer Fcns)
  - Rich dynamic responses

$$\begin{bmatrix}\Delta\varepsilon\\ \Delta\omega\end{bmatrix} = \underbrace{\begin{bmatrix}T^{\mathrm{re}}(s) & -T^{\mathrm{im}}(s)\\ T^{\mathrm{im}}(s) & T^{\mathrm{re}}(s)\end{bmatrix}}_{:=\,\underline{T}(s)}\left(\begin{bmatrix}-\Delta\rho\\ \Delta\sigma\end{bmatrix} - \underbrace{\begin{bmatrix}T^{vp}(s)\\ T^{vq}(s)\end{bmatrix}}_{:=\,\underline{T}^{v}(s)}\frac{\Delta v}{v^*}\right)$$

$$\Delta\underline{\varpi} = \underline{T}(s)\left(-\Delta\underline{\varsigma} - \underline{T}^{v}(s)\Delta v_{\mathrm{pcc}}\right)$$

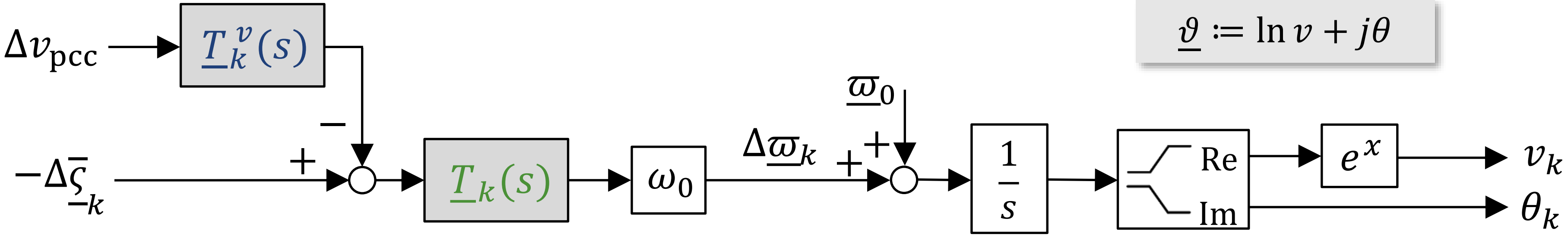
# Single-converter control setup



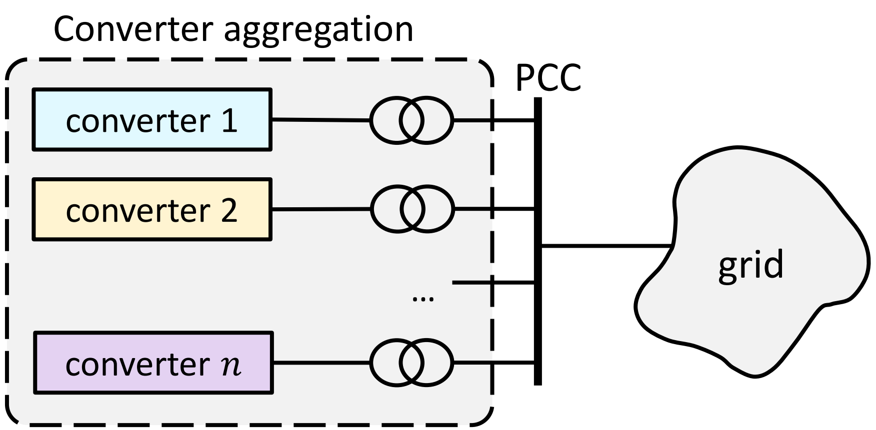

$$\Delta\underline{\varpi} = \underline{T}(s)\left(-\Delta\underline{\varsigma} - \underline{T}^{v}(s)\Delta v_{\text{pcc}}\right)$$

We choose $\underline{T}(s) = \underline{T}_{\text{des}}(s)$ and $\underline{T}^{v}(s) = \underline{T}^{v}_{\text{des}}(s)$ to provide some **desired dynamic behavior**

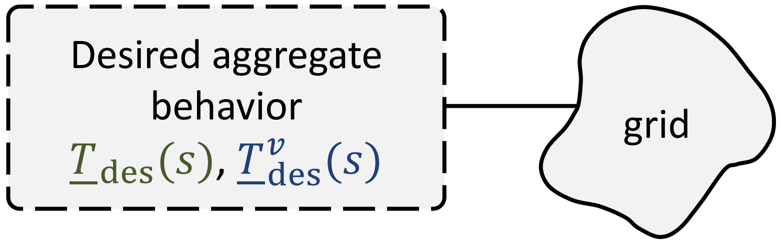
$$\underline{\vartheta} := \ln v + j\theta$$



# Multi-converter control setup (Dynamic VPP)




$\approx$

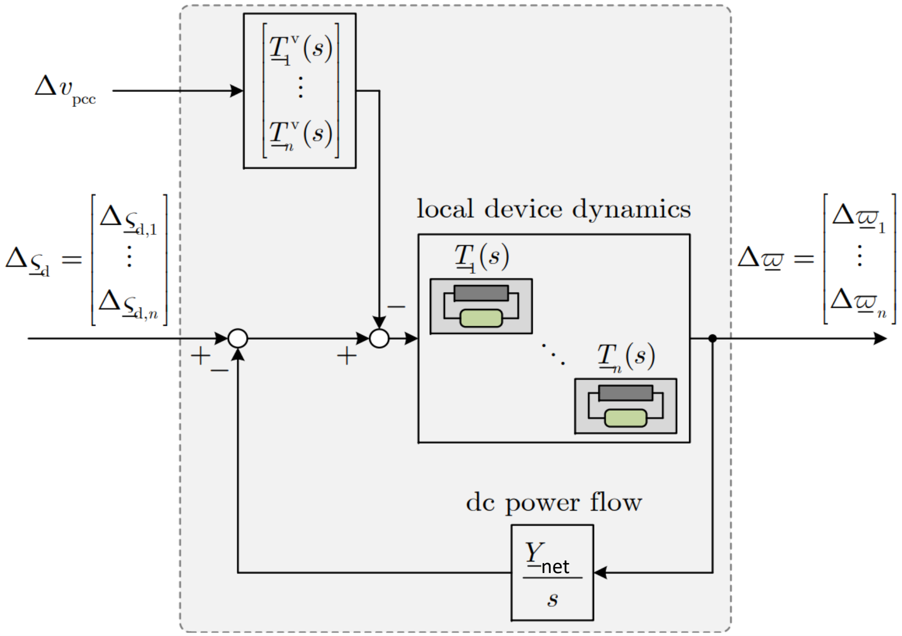



$=$

$$\Delta\underline{\varpi}_{\text{pcc}} = \underline{T}_{\text{des}}(s)\left(\Delta\underline{\varsigma}_{\text{pcc}} - \underline{T}^{v}_{\text{des}}(s)\Delta v_{\text{pcc}}\right)$$

**Aggregation conditions** for complex-frequency control

$$\left(\sum\nolimits_{k}\underline{T}_{k}(s)^{-1}\right)^{-1} \overset{!}{=} \underline{T}_{\text{des}}(s) \qquad \sum\nolimits_{k}\underline{T}^{v}_{k}(s) \overset{!}{=} \underline{T}^{v}_{\text{des}}(s)$$




$=$

$$\Delta\underline{\varpi}_{\text{pcc}} = \left(\sum\nolimits_{k}\underline{T}_{k}(s)^{-1}\right)^{-1}\left(\Delta\underline{\varsigma}_{\text{pcc}} - \sum\nolimits_{k}\underline{T}^{v}_{k}(s)\,\Delta v_{\text{pcc}}\right)$$

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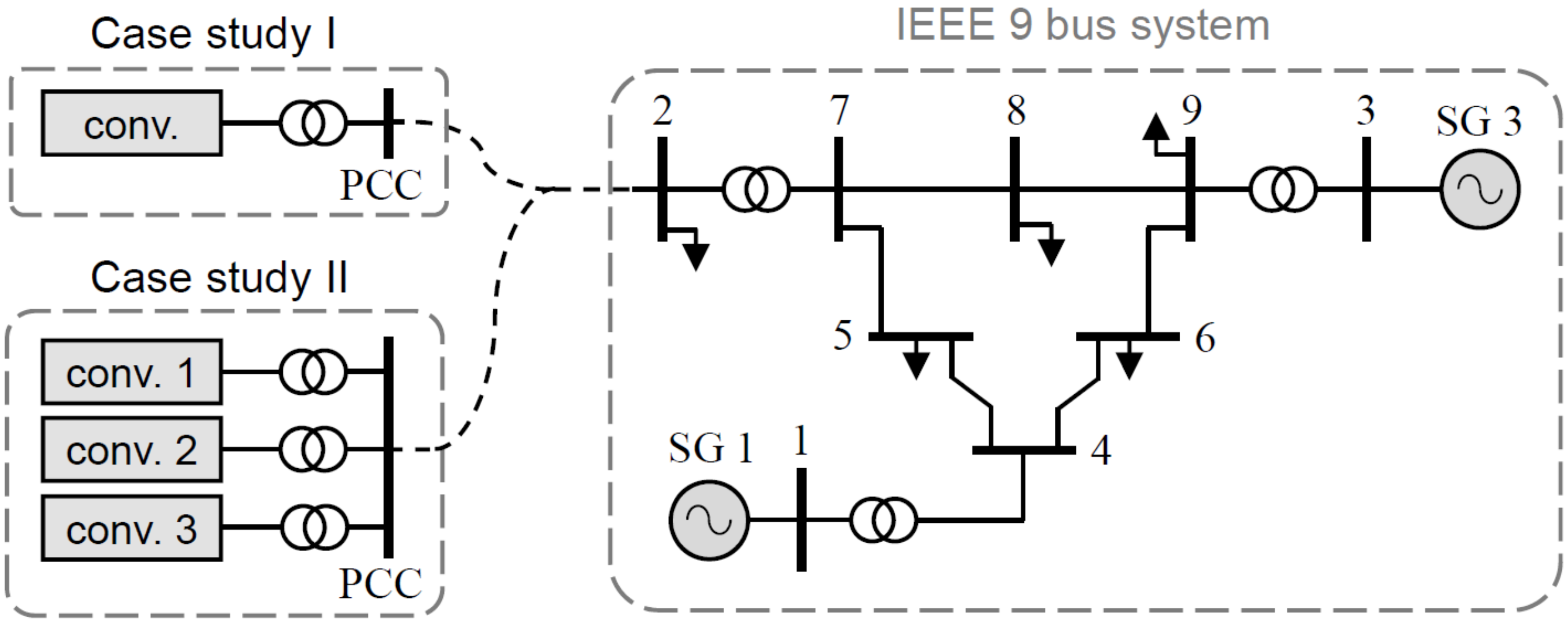
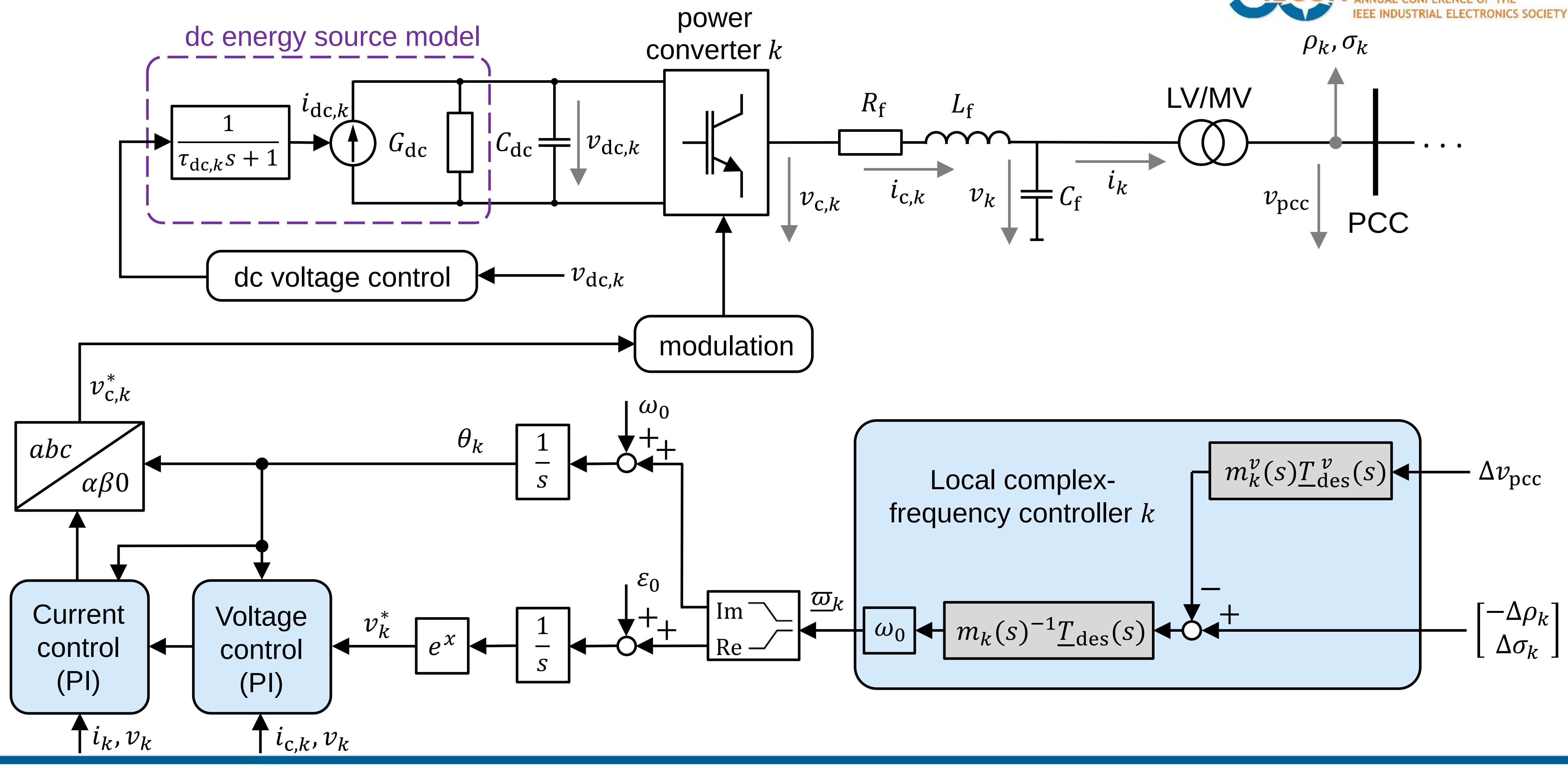
# Case study setup



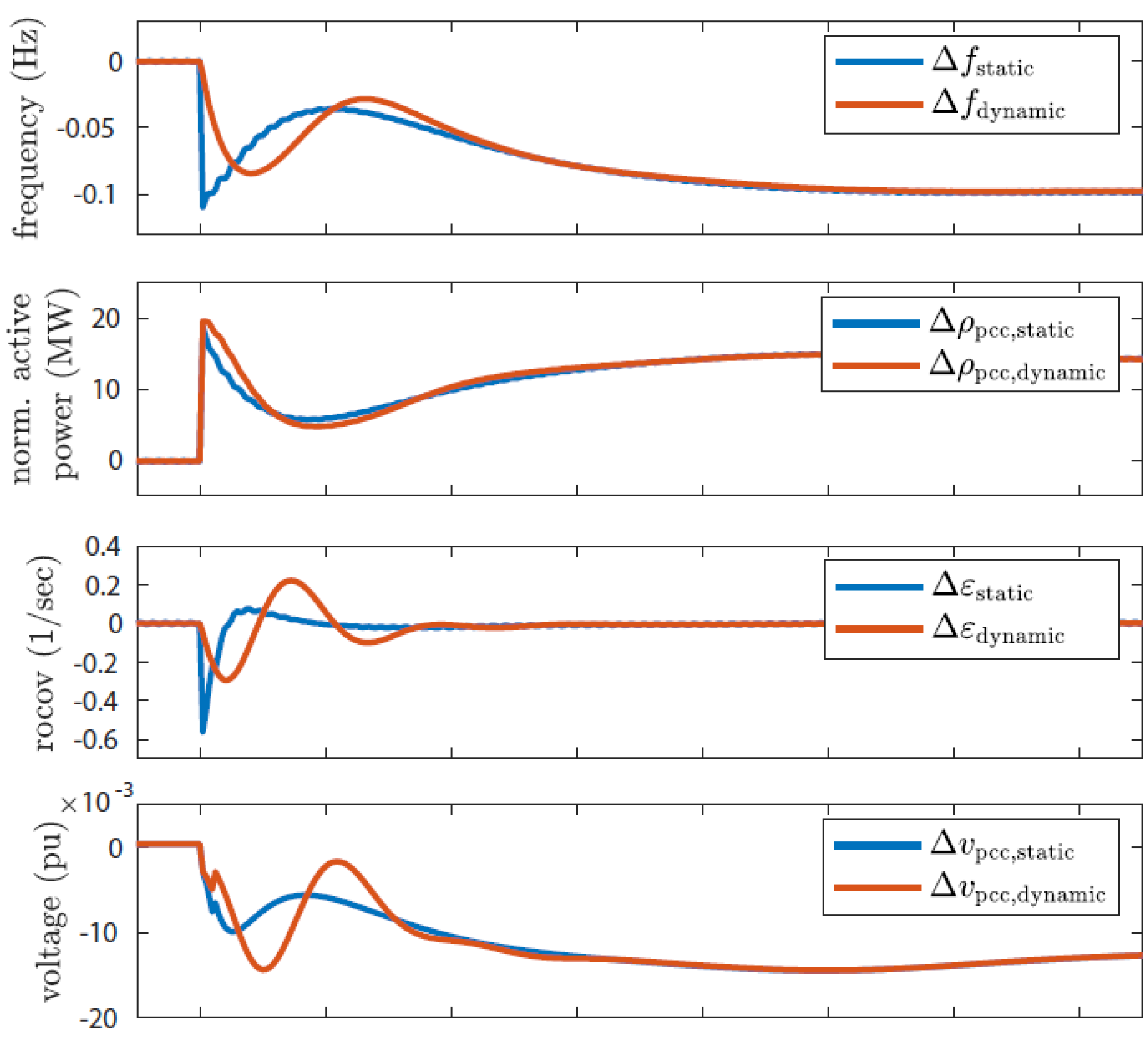



Desired response: $T_{\text{des}}(s) = \frac{e^{j\pi/4}}{2s+50}$ $\underline{T}^{v}(s) = 5$

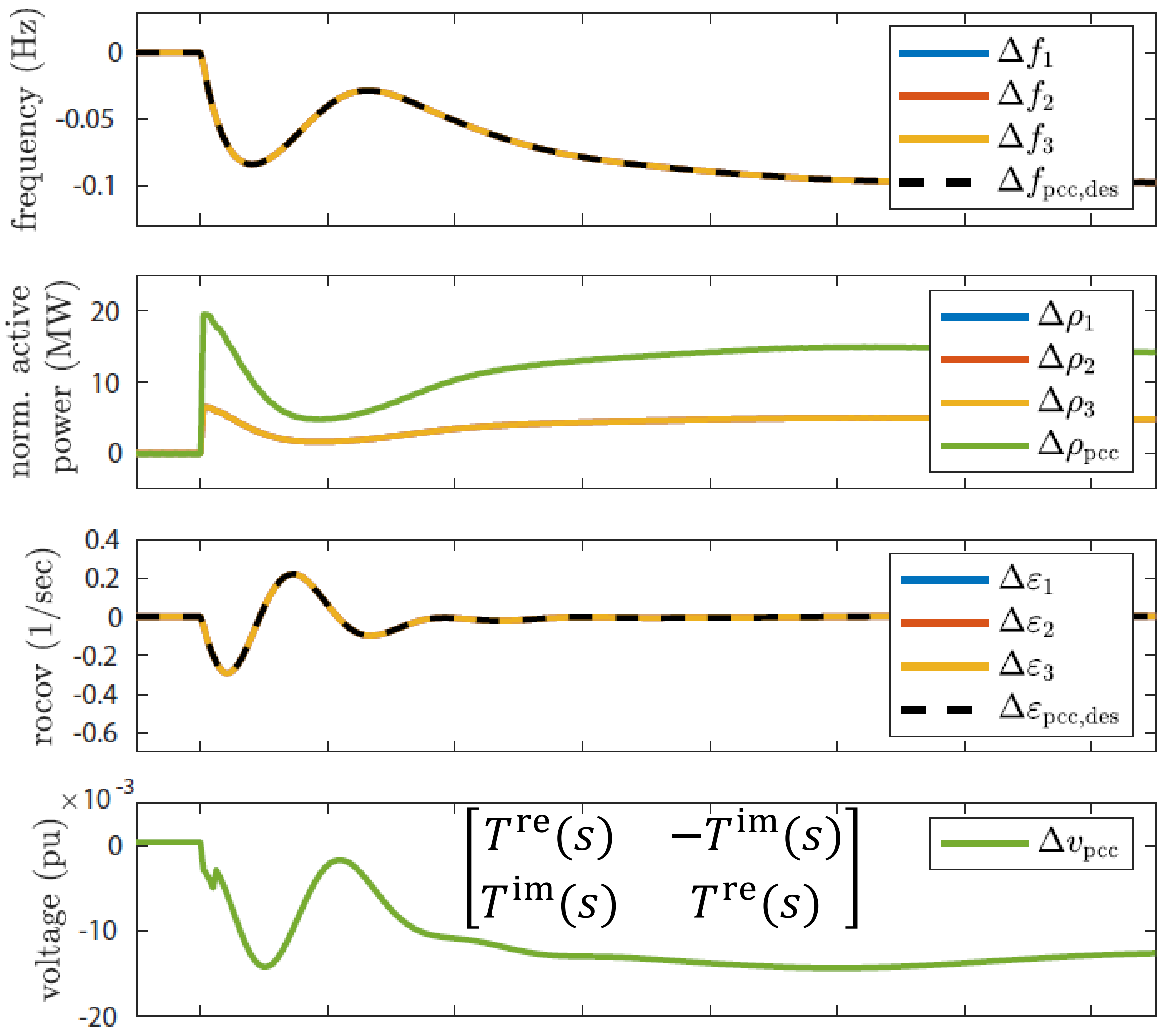
# Control structure



# Case study results




Single-converter case




$$\begin{bmatrix} T^{re}(s) & -T^{im}(s) \\ T^{im}(s) & T^{re}(s) \end{bmatrix}$$

Multi-converter case

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# Conclusion / Take-home messages

- **Complex-frequency (MIMO) grid-forming control:** It outperforms classical (SISO) ones in stability guarantees.
- **Dynamic complex-frequency control: Transfer functions** (dynamic gains) instead of static gains provide richer dynamic responses.
- **DVPPs, dynamic virtual power plants:** A good solution to coordinate multiple converters to fulfill a desired response.
- **Future work:** Stability guarantees when using dynamic (vs. static) complex-frequency control.

# Further reading

**Complex frequency**

- F. Milano, Complex frequency, IEEE TPWRS, 2022.

**Stability guarentees of complex droop control (aka. dVOC)**

- M Colombino, D Groß, JS Brouillon, F Dörfler, Global phase and magnitude synchronization of coupled oscillators with application to the control of grid-forming power inverters, IEEE TAC, 2020.
- X. He, V. Häberle, and F. Dörfler, Complex-frequency synchronization of converter-based power systems, IEEE TCNS, 2022.
- X. He, L. Huang, I. Subotić, V. Häberle, and F. Dörfler, Quantitative stability conditions for grid-forming converters with complex droop control, IEEE TPEL, 2024.
- X. He, V. Häberle, I. Subotić, and F. Dörfler, Nonlinear stability of complex droop control in converter-based power systems, IEEE L-CSS, 2023.
- X. He and F. Dörfler, Passivity and decentralized stability conditions for grid-forming converters, IEEE TPWRS, 2024.

# Acknowledgments

Thank you!

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