



清华大学 自动化系
Department of Automation, Tsinghua University

异构电力电子化电力系统分散式稳定性分析： 架构、方法及应用

Decentralized Stability Analysis of Heterogeneous Power- Electronics-Dominated Power Systems: Framework, Methods and Applications

何秀强 助理教授

Xiuqiang He, Assistant Professor

清华大学 自动化系

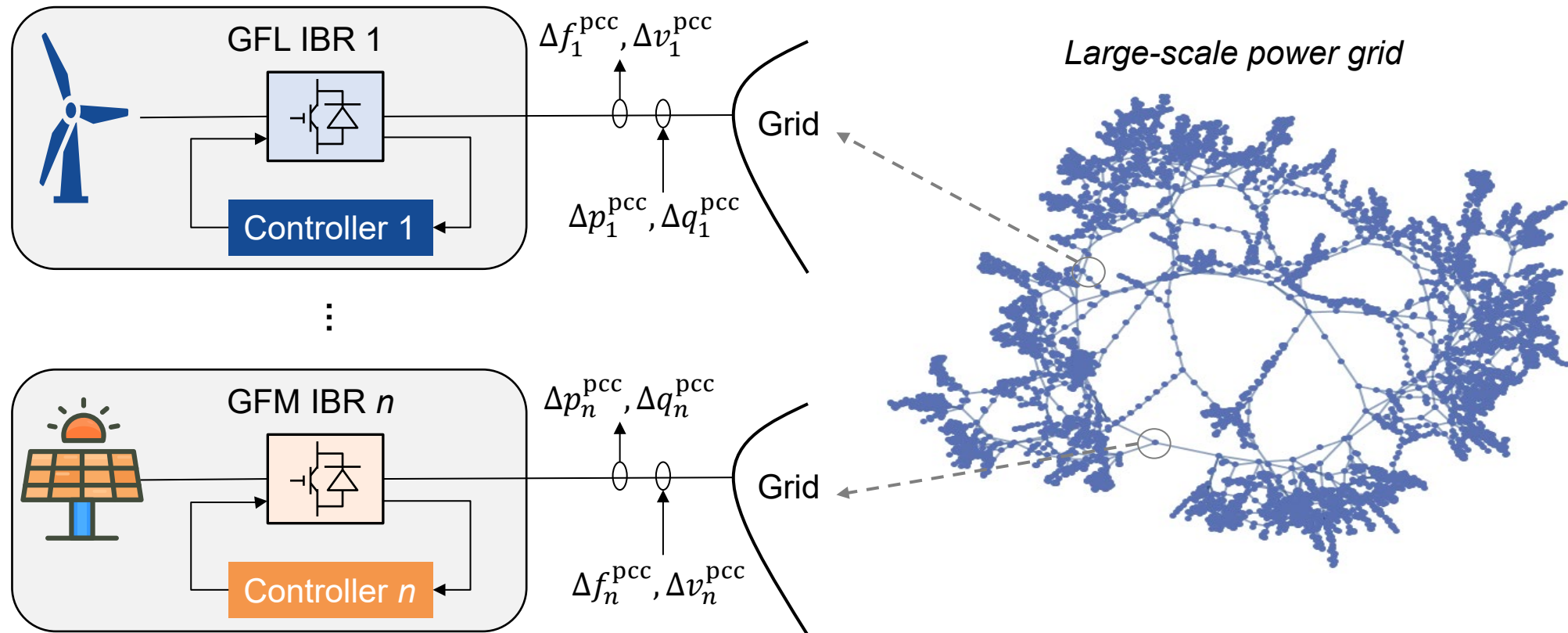
Department of Automation, Tsinghua University

xhe@tsinghua.edu.cn

Oct. 25, 2026

Heterogeneous Power Systems

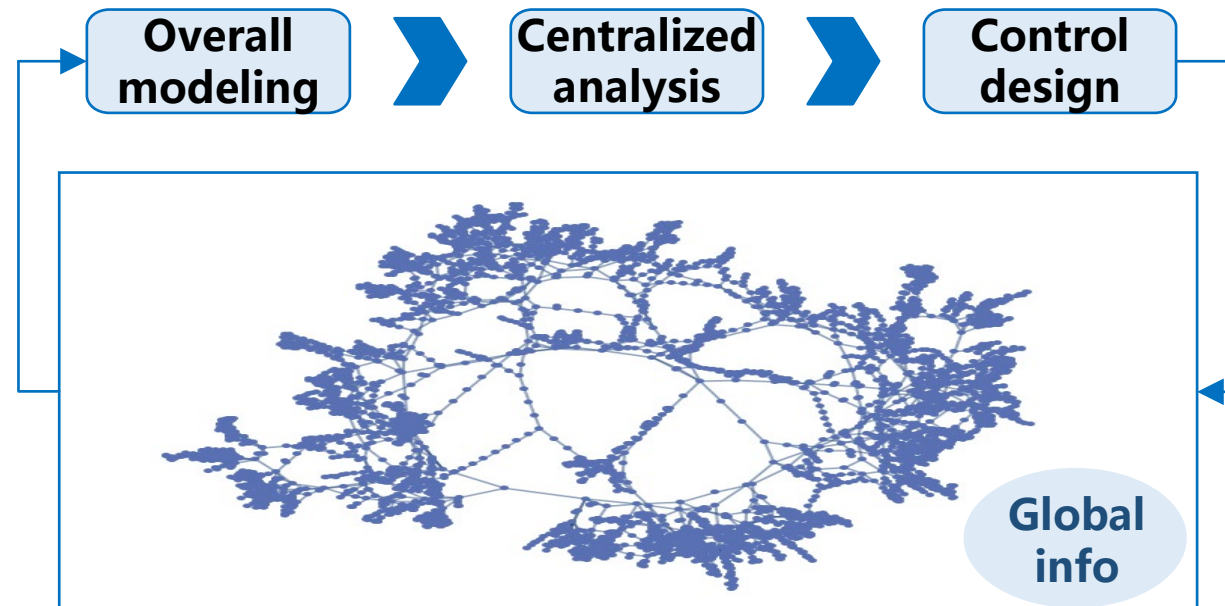
- Heterogeneous power systems comprising multiple types of devices



Centralized VS. Decentralized Framework for Stability Analysis

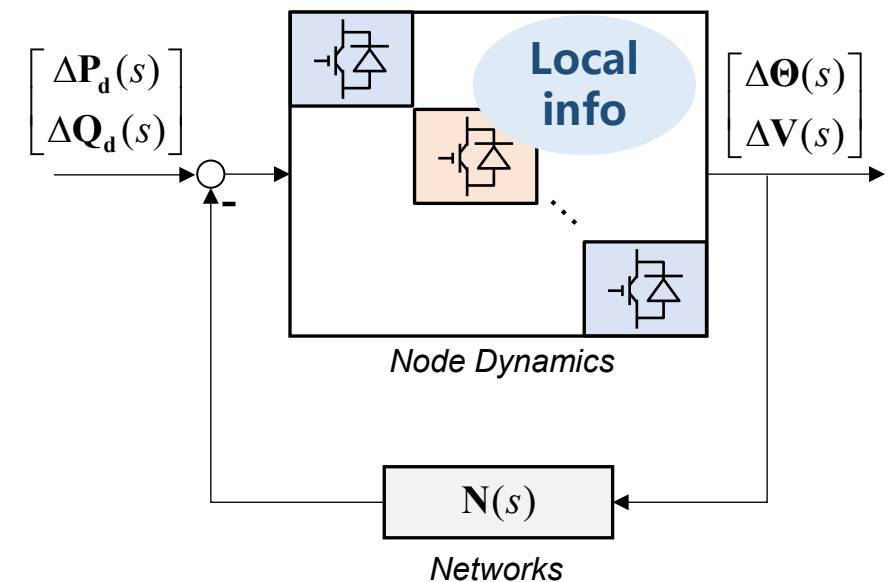
- **Centralized framework**

- Relies on global information, limited by confidentiality
- Analytical intractability
- Computational intractability
- + Low conservativeness



- **Decentralized framework**

- + Relies on local information, free of confidentiality
- + Analytical tractability
- + Computational tractability, independent of scale
- Likely high conservativeness



Contents

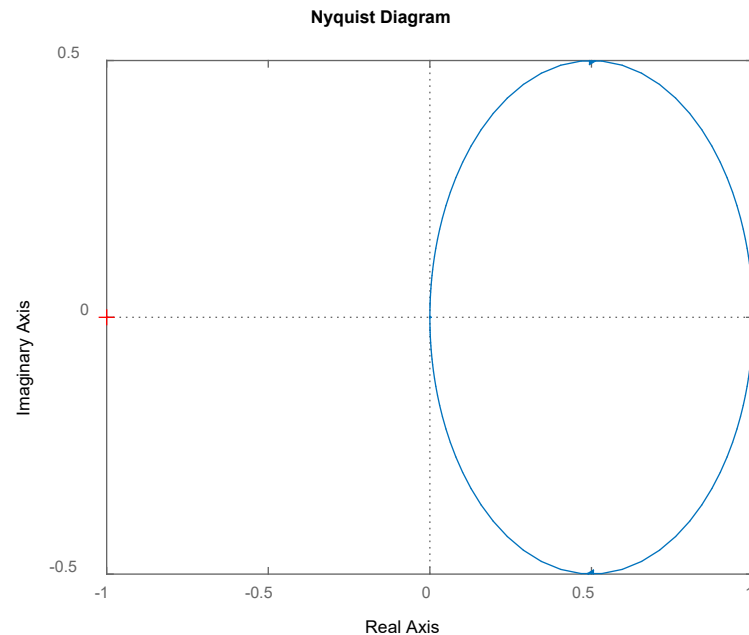
I. Typical Decentralized Stability Analysis Methods

II. Applications of Small-Signal Stability Analysis

III. Applications of Large-Signal Stability Analysis

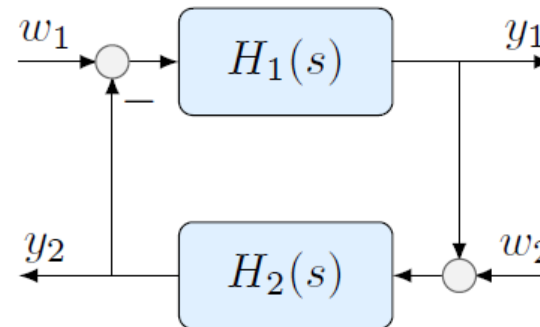
Typical decentralized method 1: Dissipativity/passivity

- **Passivity:** the Nyquist curve lies in the right half-plane.

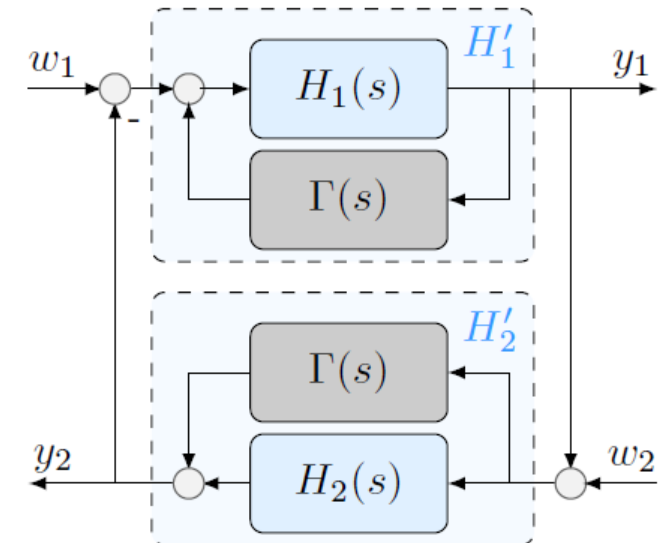


$$G(s) = \frac{s}{s+2}$$

- **Passivity-based feedback interconnection** → stability: $(-90^\circ, 90^\circ) \cup (-90^\circ, 90^\circ) \subseteq (-180^\circ, 180^\circ)$



(a) Original feedback system.

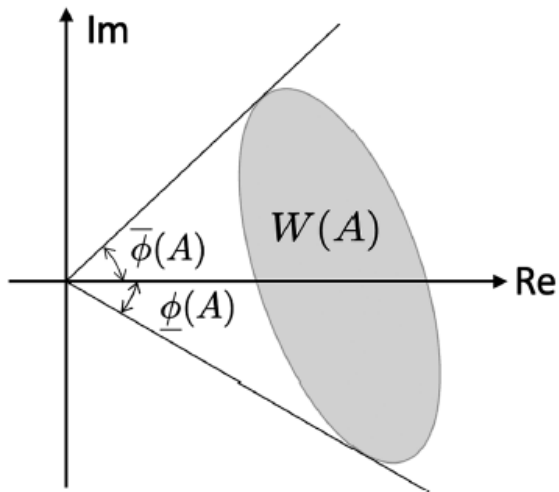


(b) Loop shifting with $\Gamma(s)$.

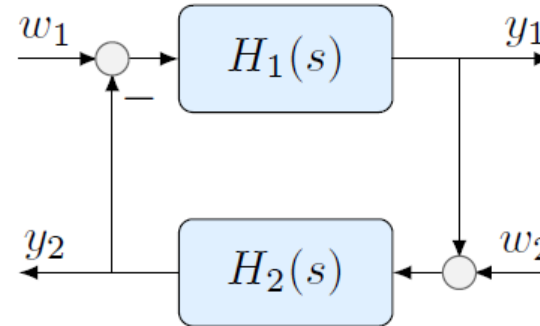
Typical decentralized method 1: generalization of linear passivity → small phase property

- **Phase of a matrix**

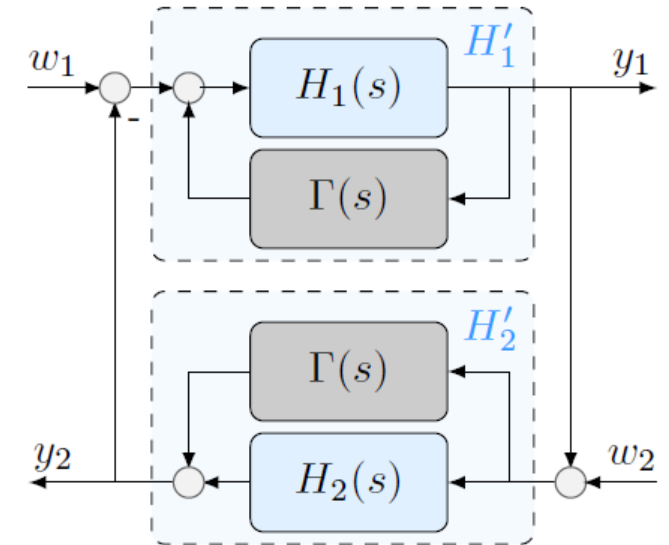
$$W(A) = \{x^*Ax : x \in \mathbb{C}^n, \|x\| = 1\},$$



- **Feedback interconnection satisfies the small-phase condition → stability:**
 $\angle H_1(j\omega) \cup \angle H_2(j\omega) \subseteq (-180^\circ, 180^\circ)$



(a) Original feedback system.

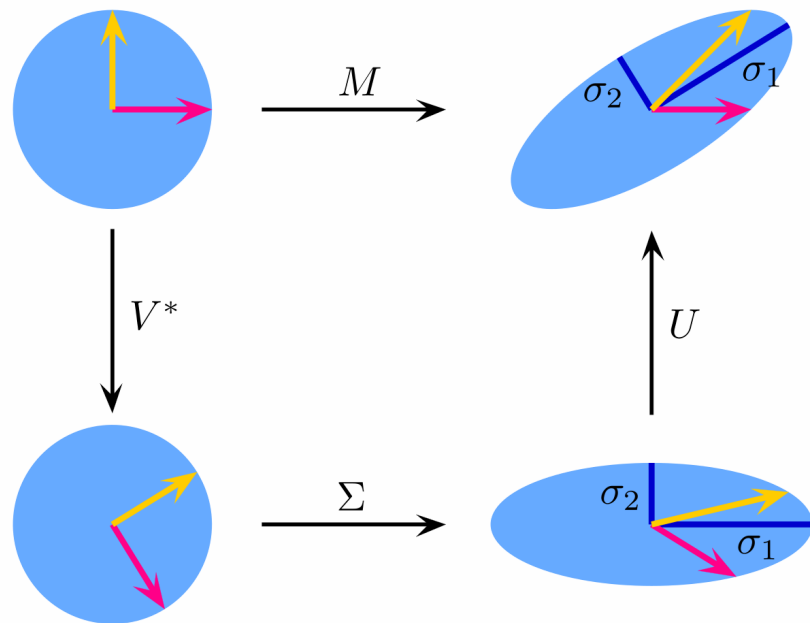


(b) Loop shifting with $\Gamma(s)$.

Typical decentralized method 2: Small gain

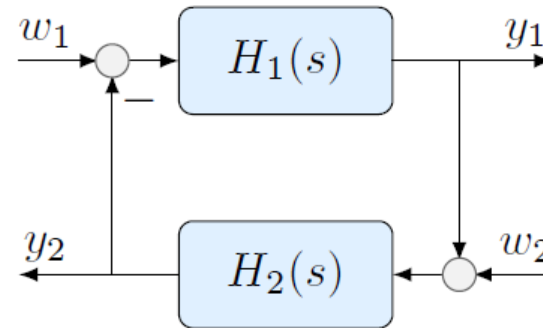
- Gains of a matrix: singular values

$$\{Mx \mid \|x\| = 1\}$$

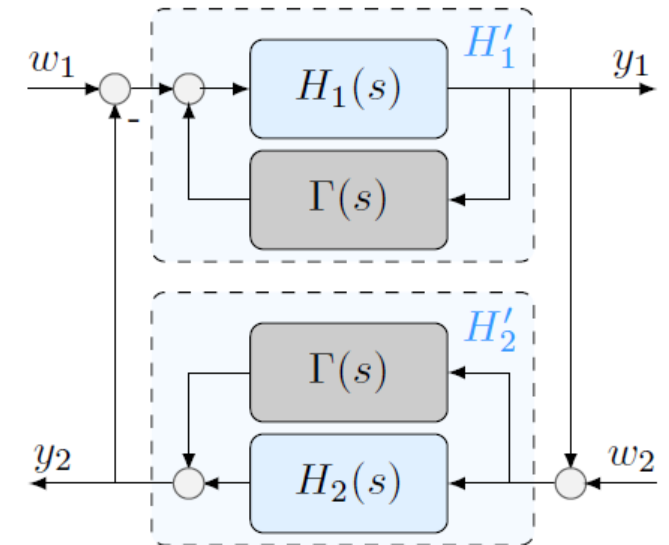


$$M = U \cdot \Sigma \cdot V^*$$

- Feedback interconnection satisfies the small-gain condition $\rightarrow$ stability:
 $\|H_1(j\omega)\| \cdot \|H_2(j\omega)\| < 1$



(a) Original feedback system.



(b) Loop shifting with $\Gamma(s)$.

Typical decentralized method 3: mixed gains and phases

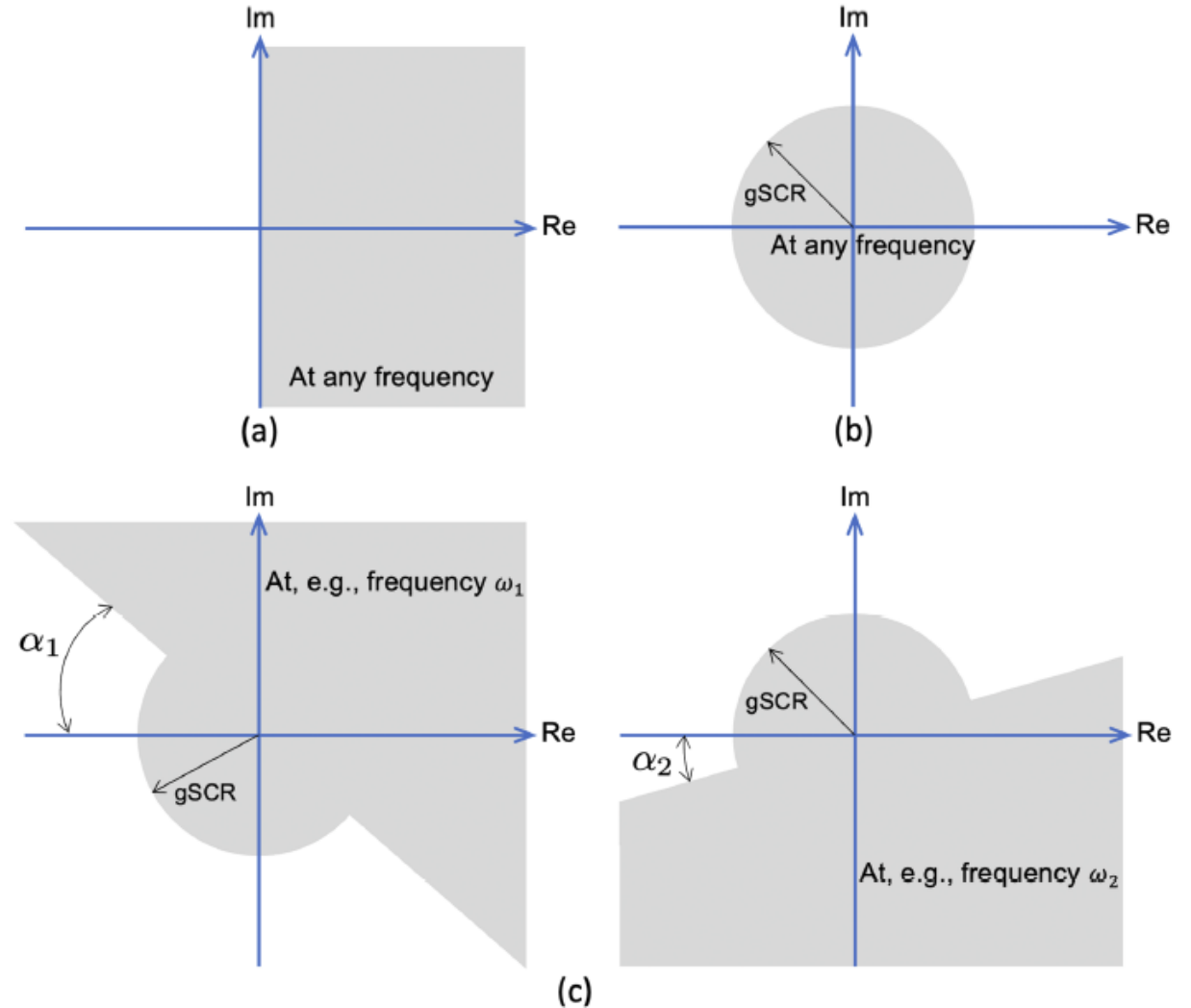
- **Mixed:** at any frequency, either the small-gain condition or the small-phase condition is satisfied.

- **A unified representation:**

$$Q(\omega) + S(\omega)^* H_1(j\omega) + H_1(j\omega)^* S(\omega) \geq \rho(\omega) H_1(j\omega)^* H_1(j\omega)$$

$$-G_2(j\omega)^* Q(\omega) G_2(j\omega) + S(\omega) G_2(j\omega) + G_2(j\omega)^* S(\omega)^* > \mu(\omega) I$$

$$\mu(\omega) + \rho(\omega) \geq 0$$



Typical decentralized method 4: SRGs

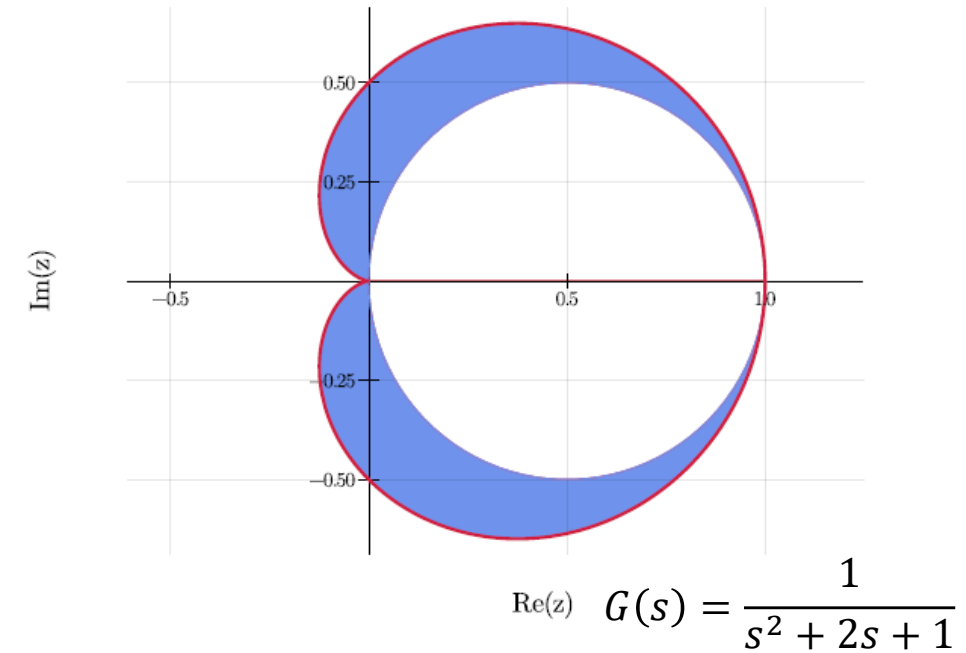
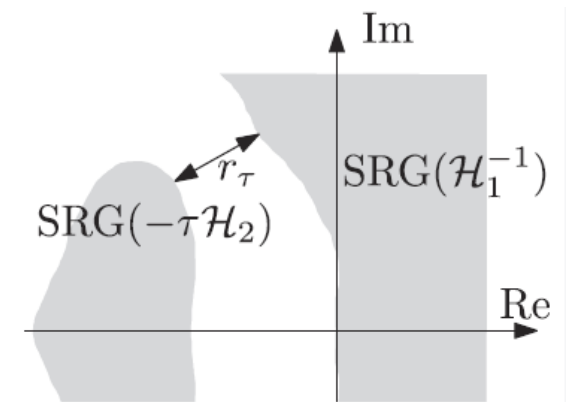
- **Scaled relative graphs (SRG)**

- SRG represents the amplification factor and phase shift imposed by the system $\{f(x, u), g(x, u)\}$:

$$\text{SRG}(f, g) = \left\{ \frac{\|y\|}{\|u\|} e^{\pm j\angle(u, y)} : y \in \text{output}, u \in \text{input}, u \neq 0 \right\}$$

- SRG is a generalization of the Nyquist plot to nonlinear systems.
- For SISO LTI systems, it can be understood as:

$$\text{SRG}(G) \leftrightarrow \{G(j\omega) : \omega \in \mathbb{R}\}$$



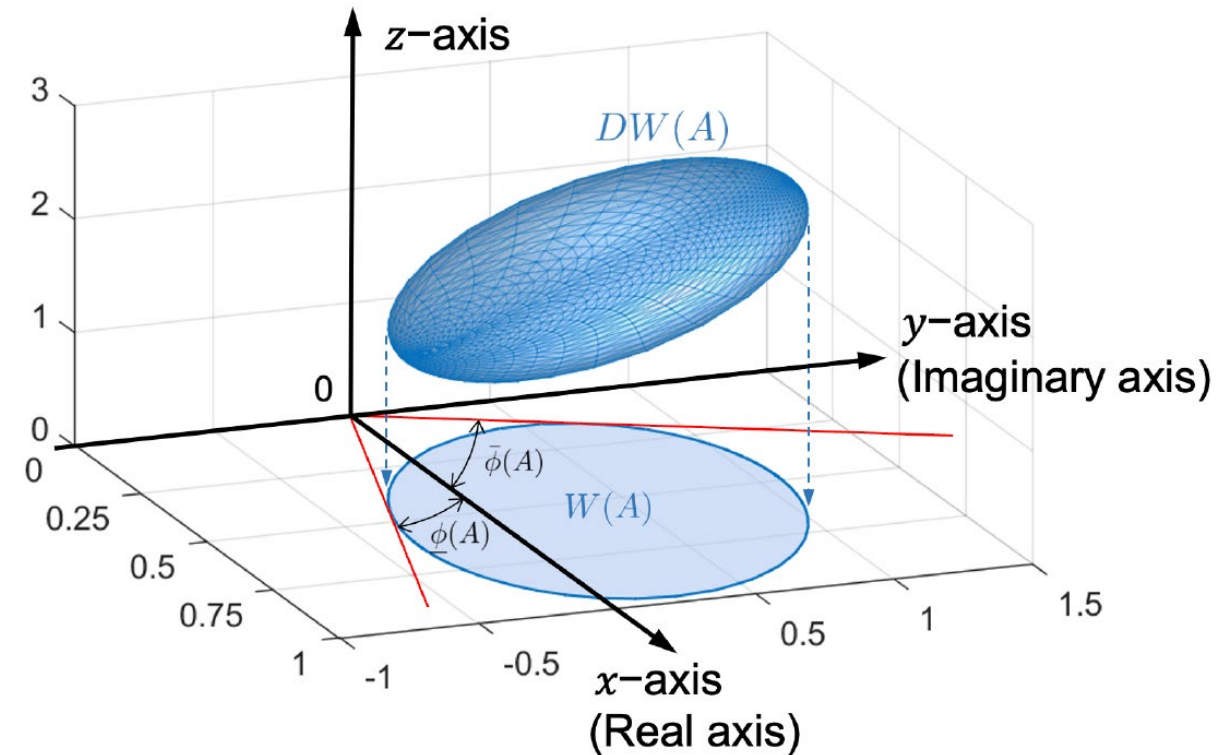
Typical decentralized method 5: DW shell

- **Davis-Wielandt (DW) shell**

- DW shell represents the output projection along the input direction and the amplification factor of matrix A :

$$DW(A) = \{(u^H A u, \|A u\|^2) : u \in \mathbb{C}^n, \|u\| = 1\}$$

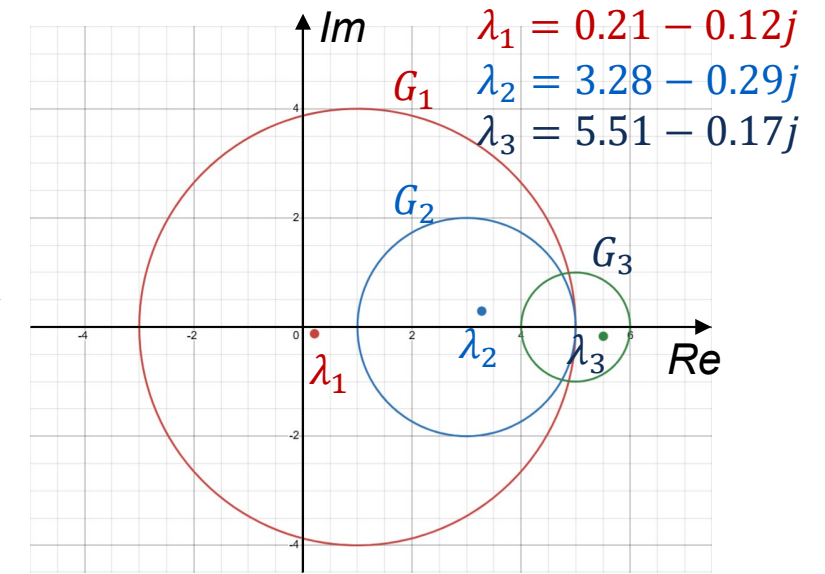
- In complex space, the DW shell contains more information than SRG (phase shift and amplification factor).
- The DW shell can provide a unified framework for passivity, small phase, small gain, and SRG of linear systems!



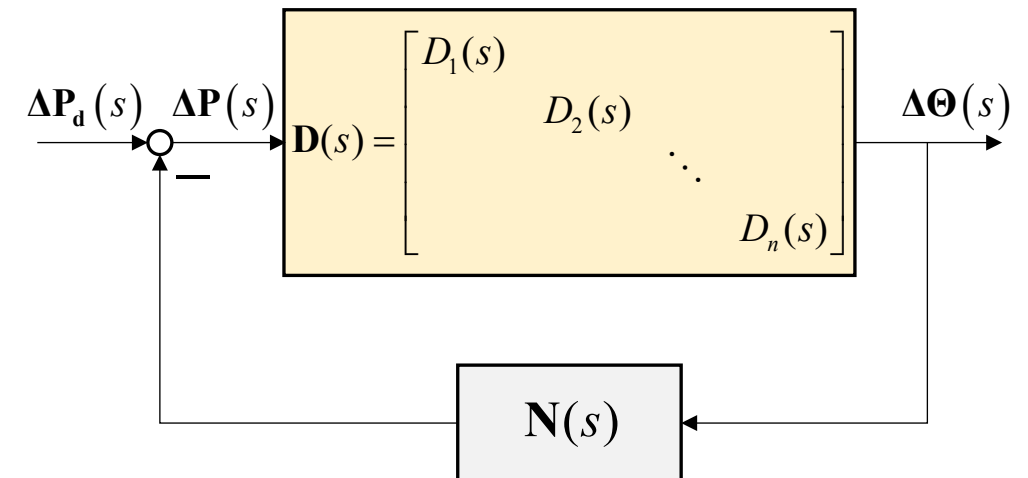
Typical decentralized method 6: Gershgorin Circle Theorem

- Gershgorin Circle Theorem

$$\mathbf{M} = \begin{bmatrix} 1 & -2 & -2 \\ -1 & 3 & j \\ 0 & -j & 5 \end{bmatrix} \xrightarrow{\text{Locate circles}} \begin{cases} G_1 = |\lambda_1 - 1| \leq 4 \\ G_2 = |\lambda_2 - 3| \leq 2 \\ G_3 = |\lambda_3 - 5| \leq 1 \end{cases} \xrightarrow{\text{Eigenvalue locations}}$$



$$G_i = \left\{ \lambda_i \in \mathbb{C} \mid \underbrace{|\lambda_i - M_{ii}|}_{\text{Mag of diagonal}} \leq \underbrace{\sum_{j \neq i} |M_{ij}|}_{\text{Sum of mag of non-diagonal}} \right\}$$



Contrasts of existing methods

	Description	Applicable systems	Interpretations
Dissipativity	Energy function, supply rate	Nonlinear, multi-port systems	★★★★★
Small gain	$\ G\ _{\infty}$, gain product	Nonlinear, uncertain systems	★★★★☆
SRG	Amplification factor, phase shift	Nonlinear, multi-port systems	★★★☆☆
DW Shell	Complex inner product, amplification factor	Linear, multi-port systems	★★★☆☆
Gershgorin	Eigenvalue inclusion circles	LTI systems	★★★★☆

Contents

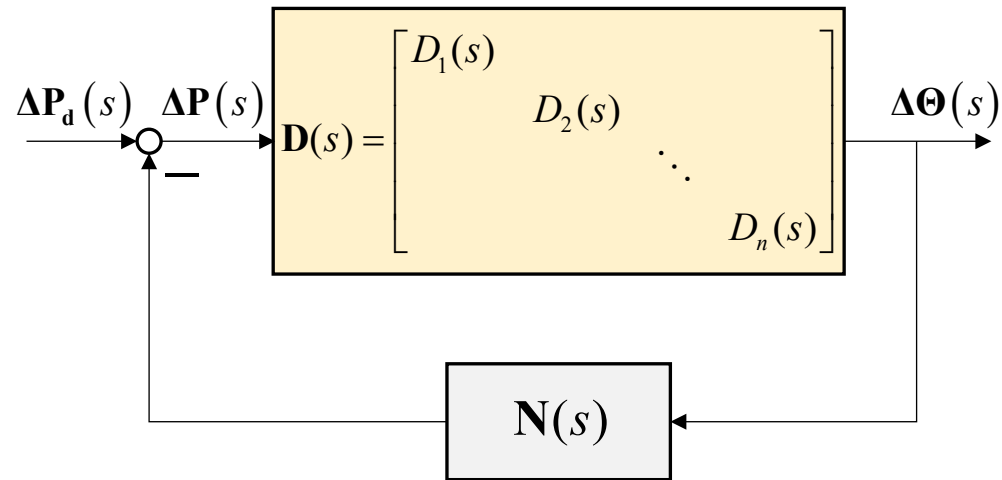
I. Typical Decentralized Stability Analysis Methods

II. Applications of Small-Signal Stability Analysis

III. Applications of Large-Signal Stability Analysis

Small-signal decentralized stability analysis

- **From Gershgorin theorem to local gain condition**



- **Chara. equ.** $\det(\mathbf{I} + \mathbf{D}(s)\mathbf{N}(s)) = 0$

$$\Rightarrow \det(\mathbf{D}(s)) \det \left(\underbrace{(\mathbf{D}(s))^{-1} + \mathbf{N}(s)}_{\mathbf{R}(s)} \right) = 0$$

- **Local gain condition:** $|\mathbf{R}_{ii}(s)| > \sum_{i \neq j} |\mathbf{R}_{ij}(s)|, \forall s \in \Gamma$

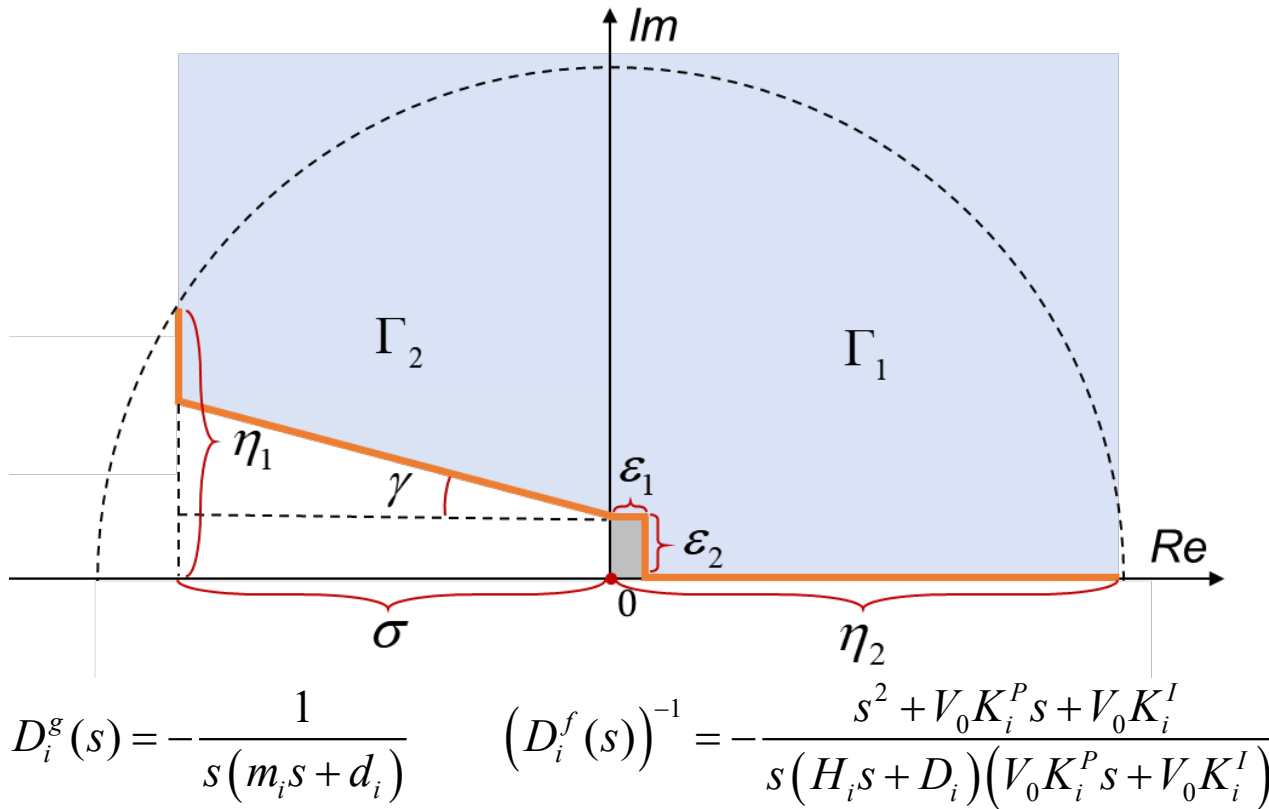
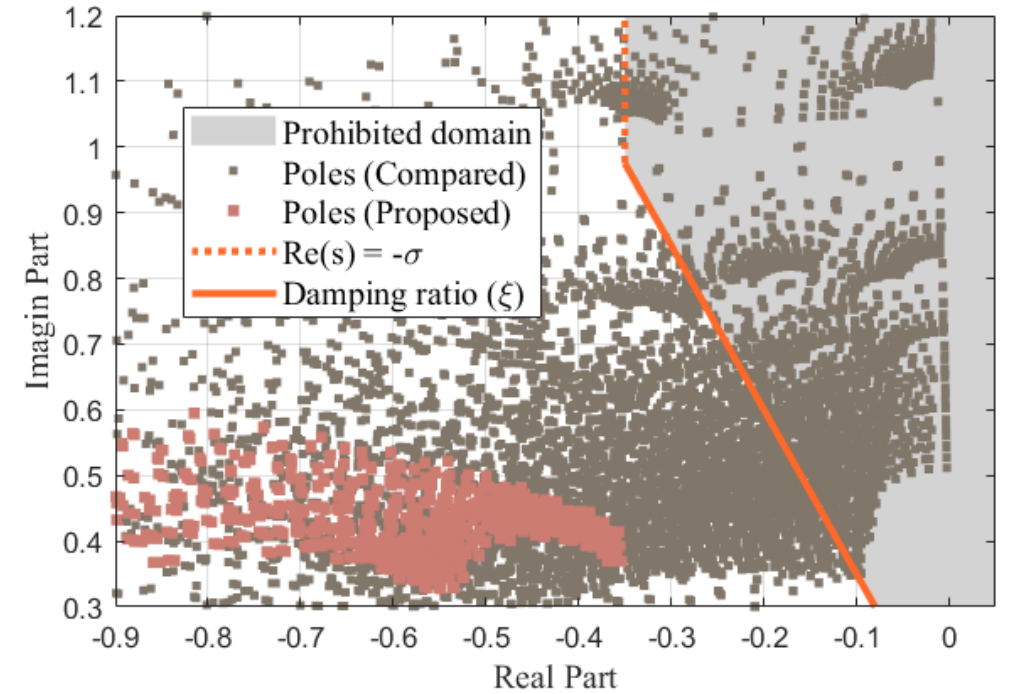
$$\Rightarrow \underbrace{\left| (D_i(s))^{-1} + N_{ii}(s) \right|}_{\text{Local gain}} > \underbrace{\sum_{j \neq i} |N_{ij}(s)|}_{\text{Network coupling}}, \forall s \in \Gamma$$

$$\Rightarrow \det(\mathbf{I} + \mathbf{D}(s)\mathbf{N}(s)) \neq 0, \forall s \in \Gamma$$

Local gain dominance: requires local dynamics to be dominant relative to coupling dynamics, so as to resist the influence of network coupling.

Use case

- Use decentralized methods to guarantee the closed-loop modal damping ratio



$$\left| (D_i(s))^{-1} + N_{ii}(s) \right| > \sum_{j \neq i} |N_{ij}(s)|, \forall s \in \Gamma$$

Contents

I. Typical Decentralized Stability Analysis Methods

II. Applications of Small-Signal Stability Analysis

III. Applications of Large-Signal Stability Analysis

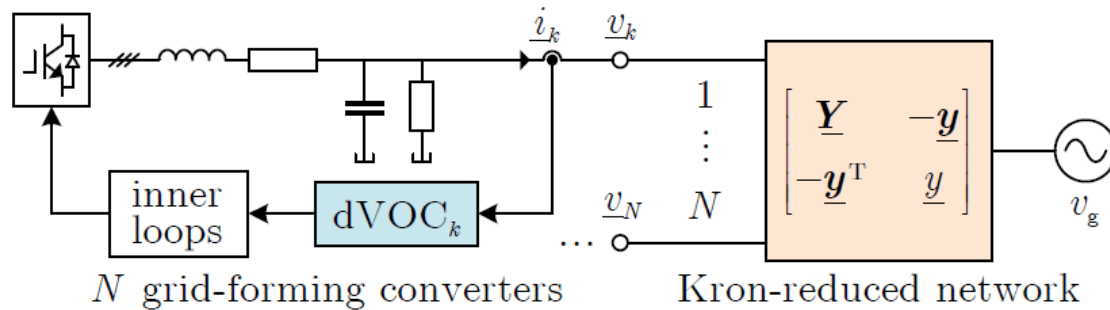
Large-signal passivity analysis

- Grid-forming dVOC control is passive**

$$\dot{\underline{v}}_k = j\omega_\Delta \underline{v}_k + \eta_k e^{j\varphi} \left(\frac{p_k^* - jq_k^*}{v_k^{*2}} \underline{v}_k - \underline{i}_k \right) + \eta_k \alpha_k \frac{v_k^{*2} - |\underline{v}_k|^2}{v_k^{*2}} \underline{v}_k$$

↓ Node's passivity index

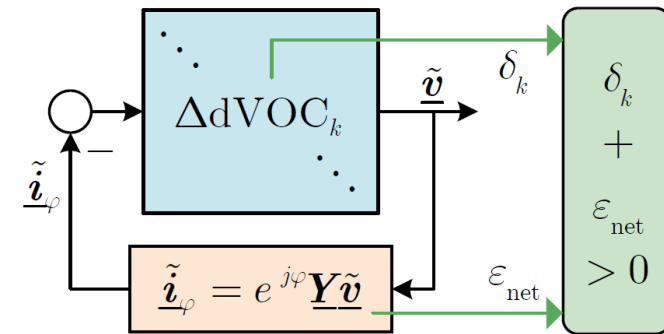
$$\delta_k := -\operatorname{Re}\left\{e^{j\varphi} \frac{p_k^* - jq_k^*}{v_k^{*2}}\right\} - \alpha_k + \alpha_k \frac{1}{2v_k^{*2}} |\underline{v}_{s,k}|^2.$$



$$\underline{i} = \underline{Y} \underline{v} - \underline{y} v_g$$

↓ Network's passivity index

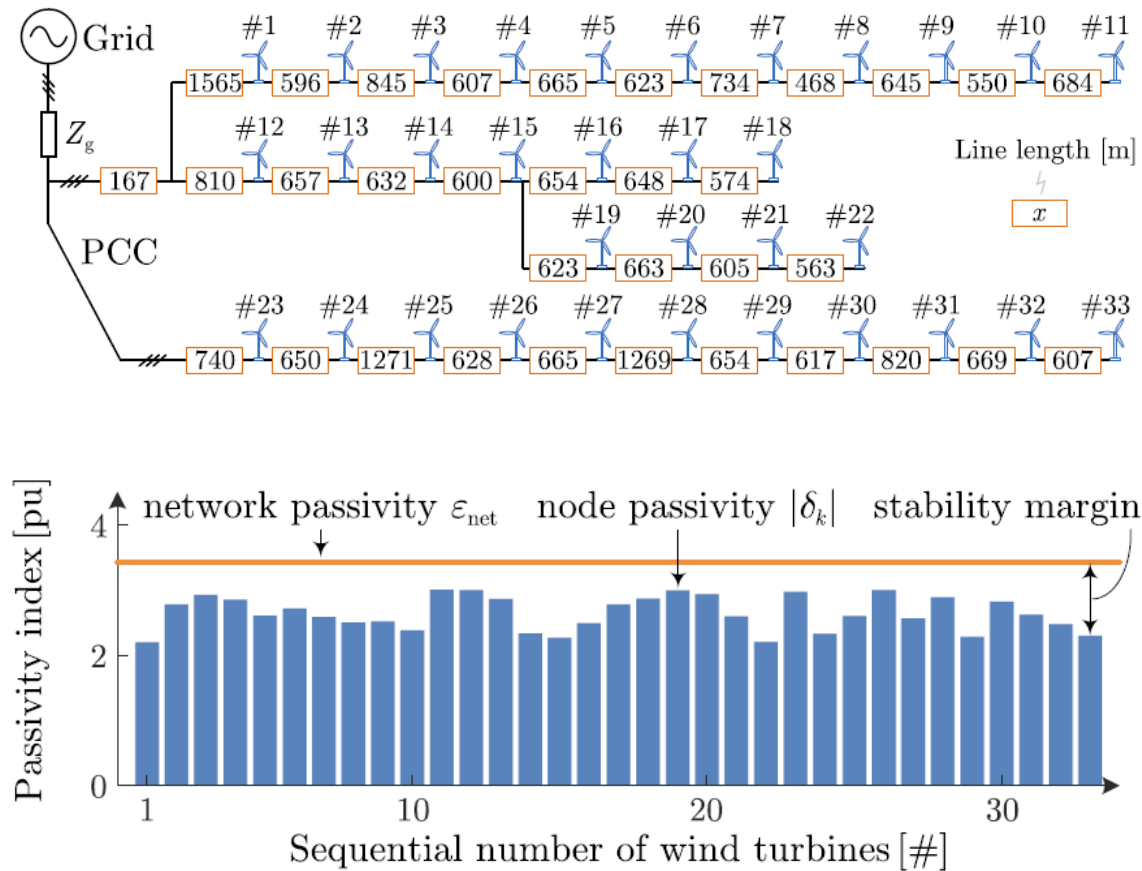
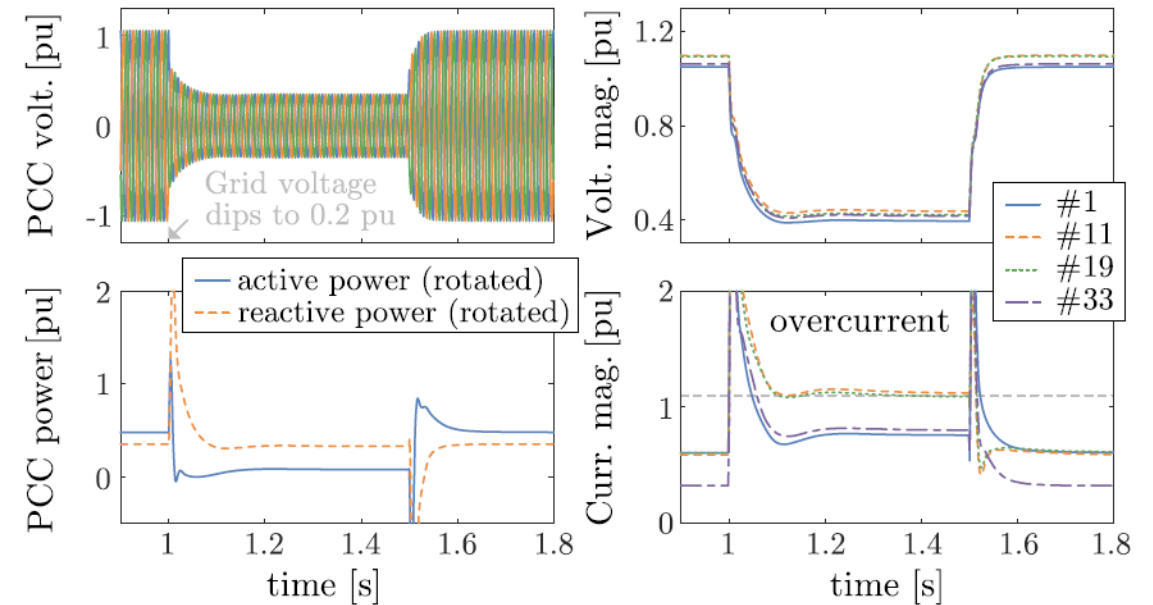
$$\varepsilon_{\text{net}} := \lambda_{\min}(\operatorname{Re}\{e^{j\varphi} \underline{Y}\})$$



↓ Decentralized stability condition

Use case

- Multi-inverter grid-connected stability is guaranteed through decentralized indices



Decentralized stability specification: $|\delta_k| < \varepsilon_{\text{net}}$

Conservatism: failure to satisfy the criterion does not necessarily imply instability.

Take aways

1. **Decentralized stability analysis** is a new approach to addressing massive heterogeneous power-electronics-dominated power systems.
2. For linear systems, **the DW shell** provides a unified analysis method.
3. For nonlinear systems, **dissipativity/passivity analysis** is the most commonly used and interpretable method, but issues such as its analytical tractability and conservatism need to be overcome.



清华大学 自动化系
Department of Automation, Tsinghua University

Thank you!

Decentralized Stability Analysis of Heterogeneous Power-Electronics-Dominated Power Systems: Framework, Methods and Applications

Xiuqiang HE

Department of Automation

Oct. 25, 2026