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Quantifying Stability Conditions for Grid-Forming and Grid-Following Heterogeneous Power Systems

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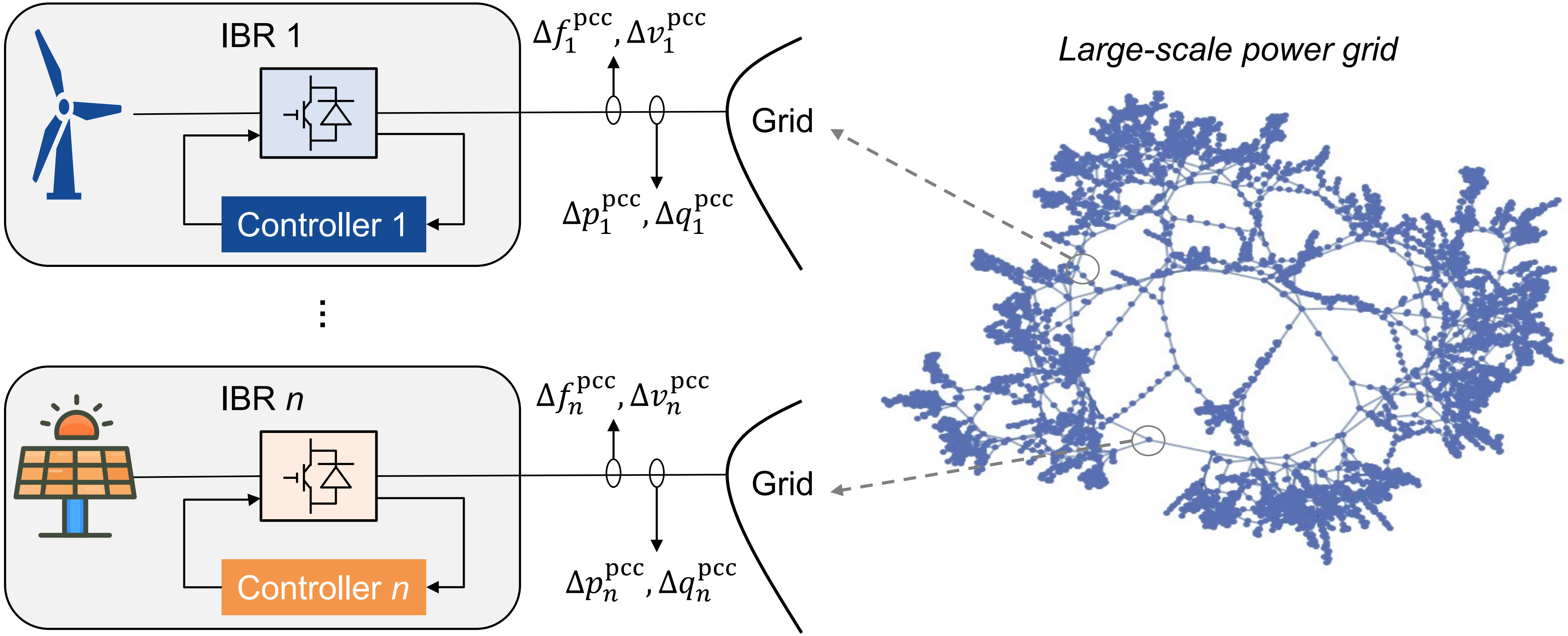
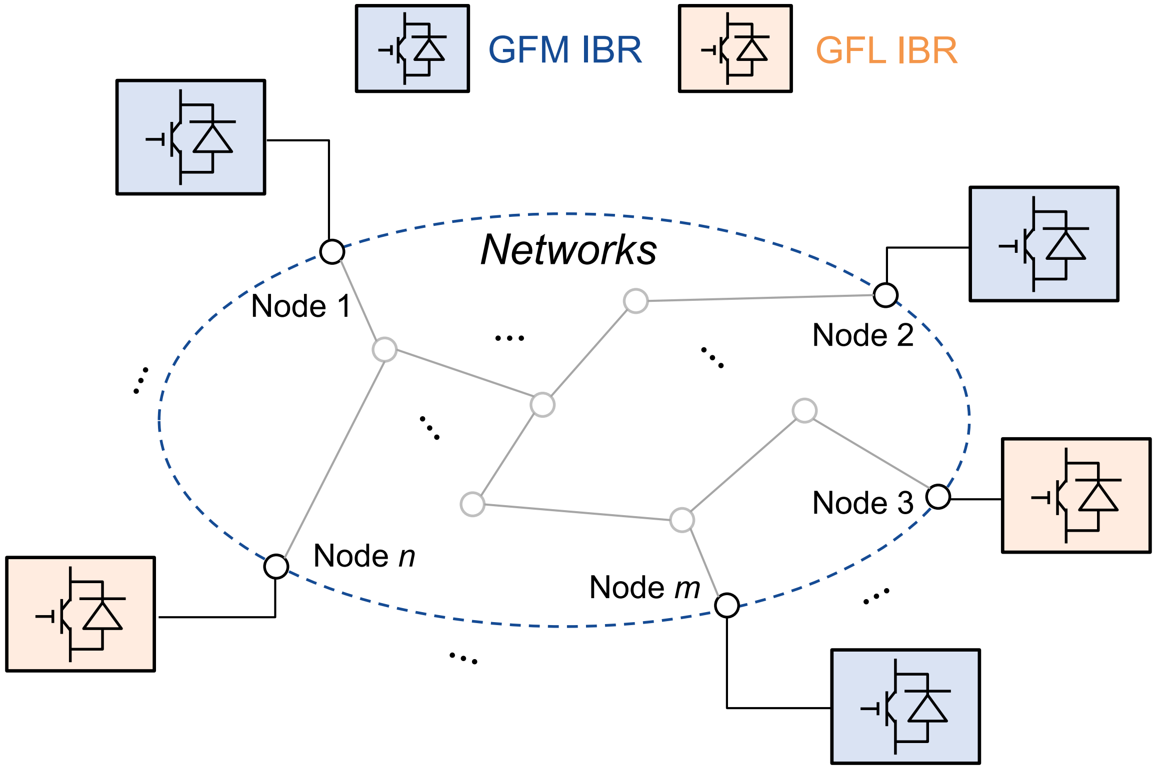
Quantifying Stability Conditions for IBRs



- Inverter-based resources (IBRs)-dominated power systems suffer from many stability issues
- Quantifying stability conditions can provide guidelines for developing IBRs' dynamic response specifications (grid codes) and control design

Heterogeneous Power Systems



- GFM controls:

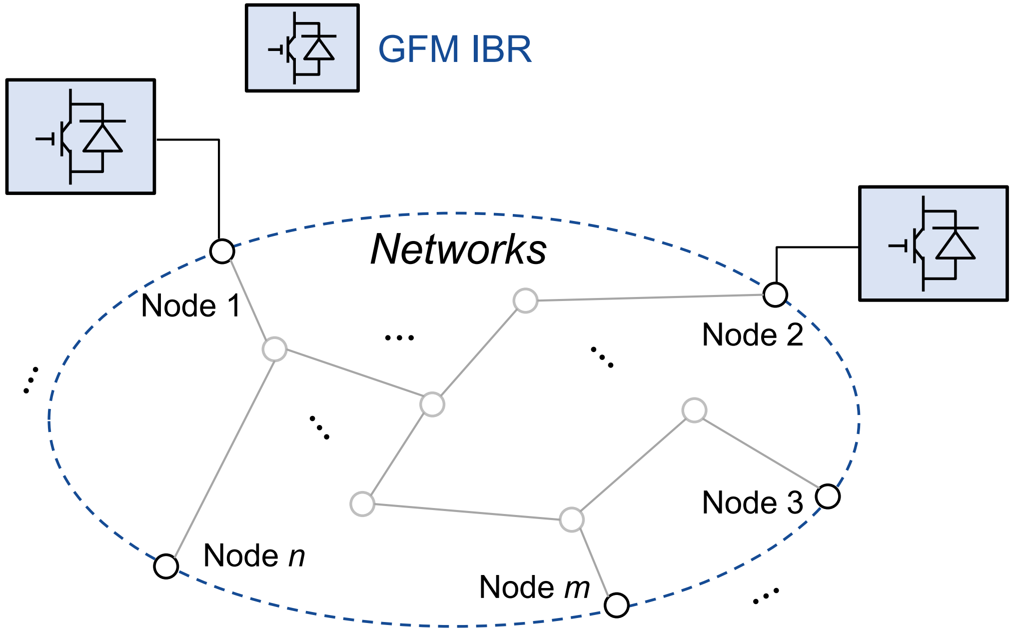
$$\begin{cases} D_i^{g,p}(s) = \dfrac{\Delta\theta_i^g(s)}{\Delta P_i^g(s)} \\ D_i^{g,q}(s) = \dfrac{\Delta v_i^g(s)}{\Delta Q_i^g(s)} \end{cases}$$

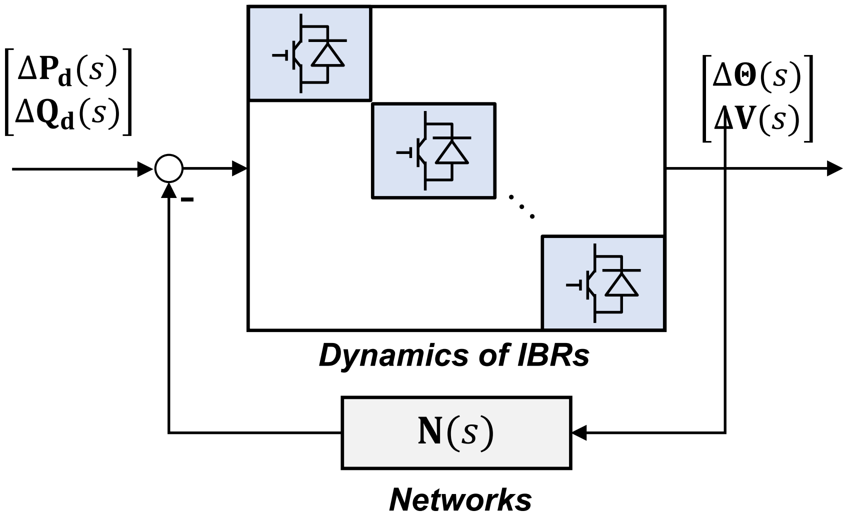
- GFL controls

$$\begin{cases} D_i^{f,p}(s) = \dfrac{\Delta P_i^f(s)}{\Delta\theta_i^f(s)} \\ D_i^{f,q}(s) = \dfrac{\Delta Q_i^f(s)}{\Delta v_i^f(s)} \end{cases}$$

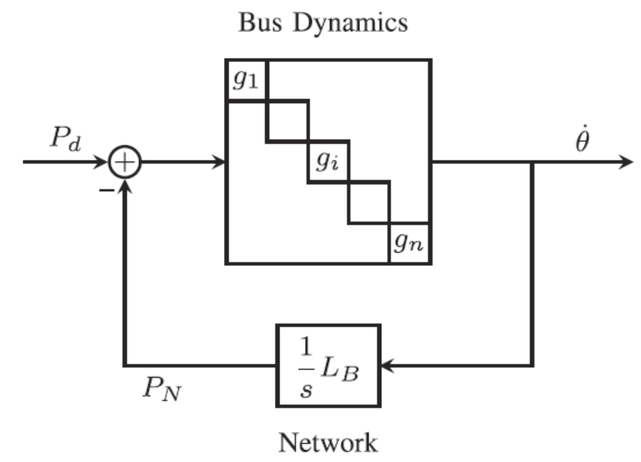
- Grid-forming (GFM) control and grid-following (GFL) control are two typical control modes
- Future IBR-dominated power systems will be **GFM-GFL-mixed**

Centralized Analysis Framework

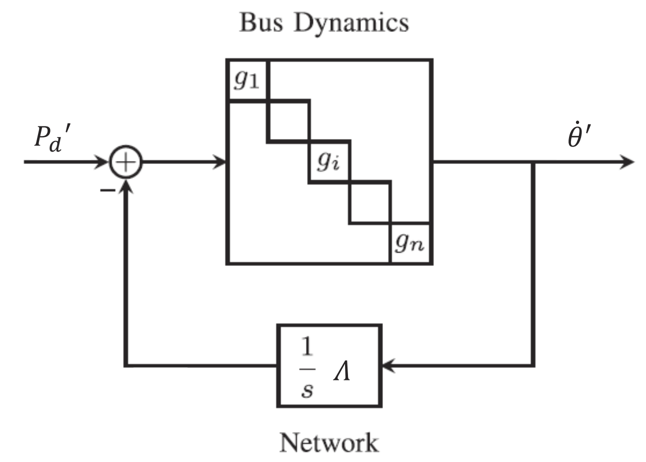




- A decoupling analysis approach based on eigen-decomposition [1]

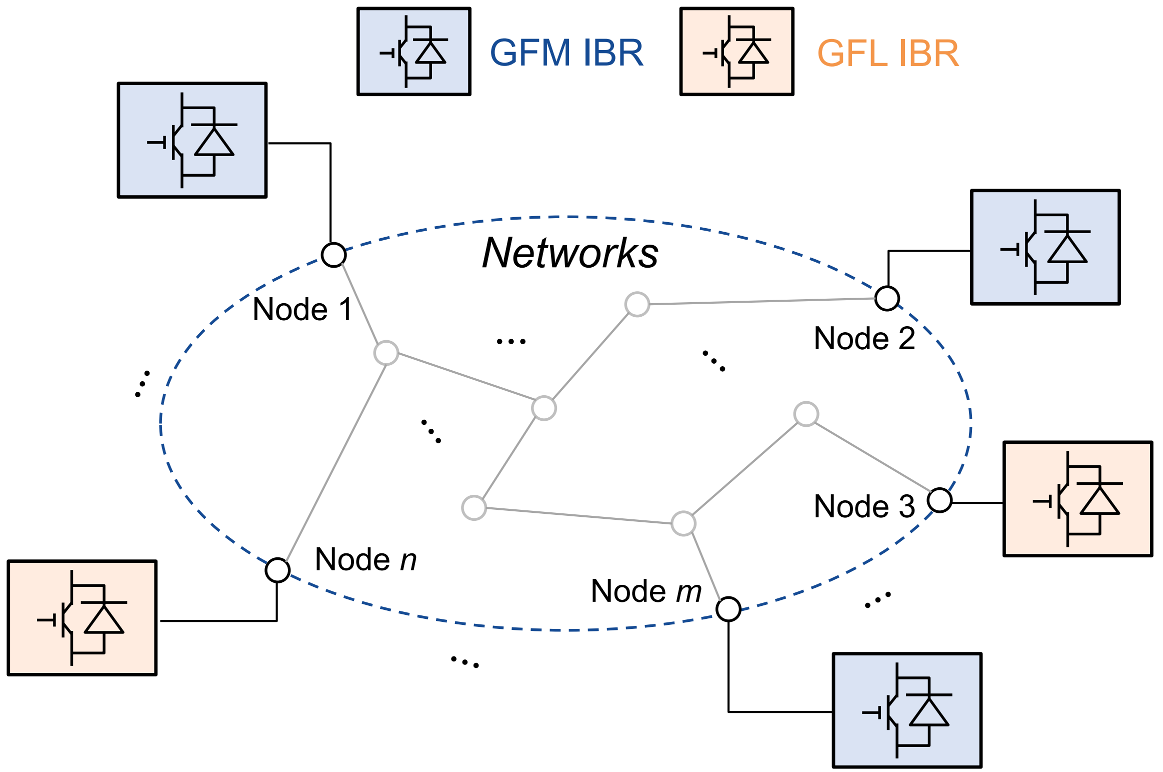


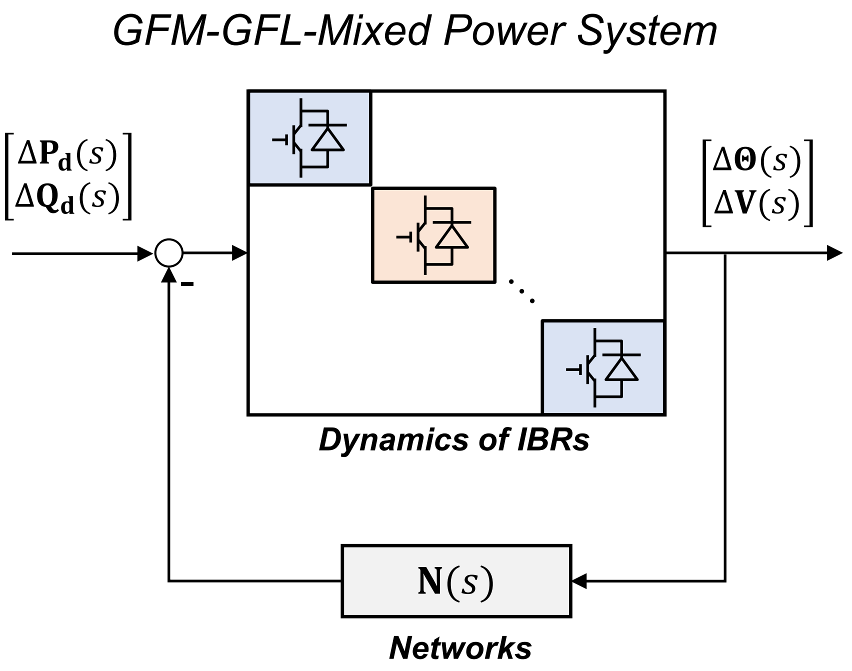
$$W^{-1}L_BW = \Lambda$$



[1] Jiang, Y., Pates, R., & Mallada, E. (2020). Dynamic droop control in low-inertia power systems. *IEEE TAC*, *66*(8), 3518-3533.

Decentralized Analysis Framework





Why Decentralization? ***Massiveness, Heterogeneity, Model-agnosticism***

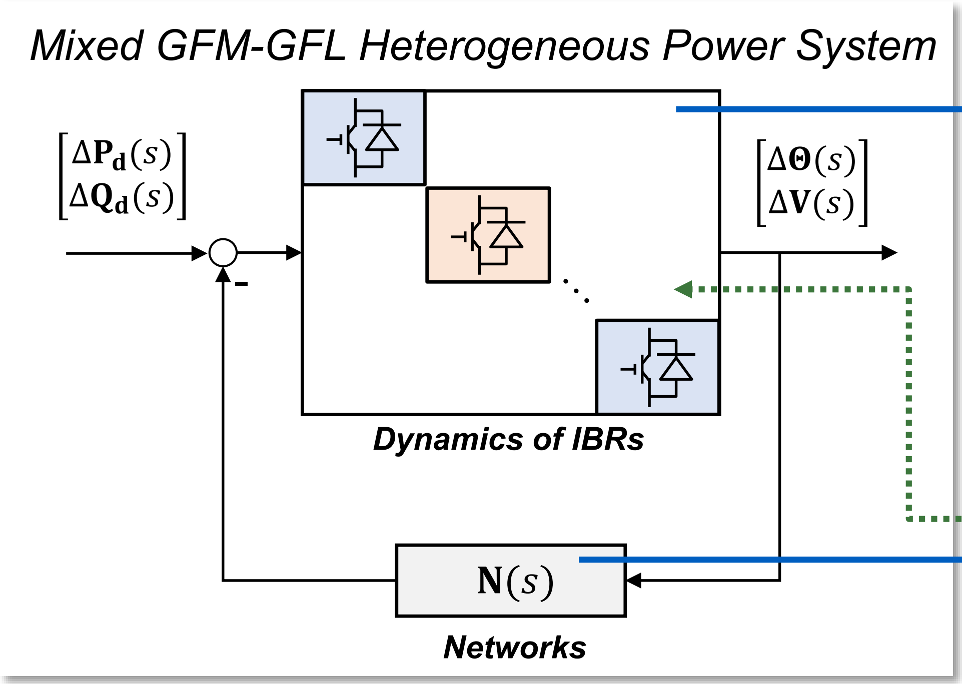
- Passivity (with multipliers or loop shifting)
- Small gains (quite conservative)

Only stability (w/o ensuring damping ratio)

Today → A new decentralized approach

- Stability
- Damping ratio (pole distribution)

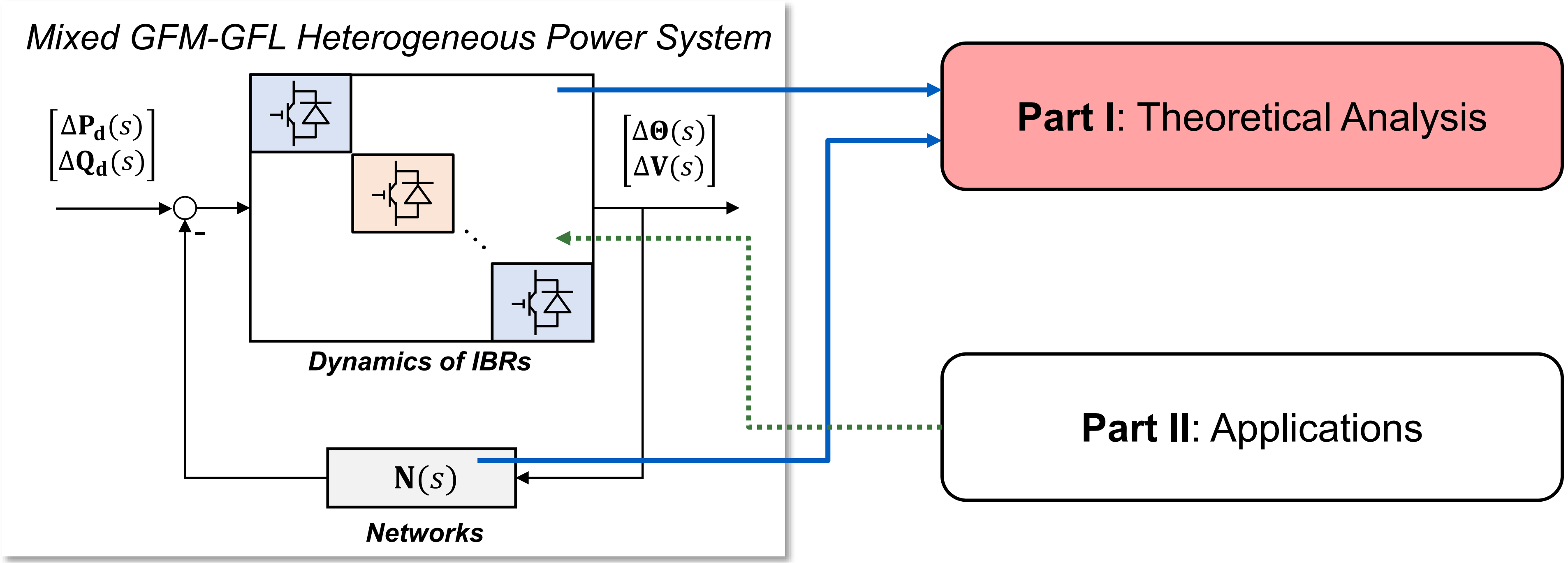
Outline



Part I: Theoretical Analysis

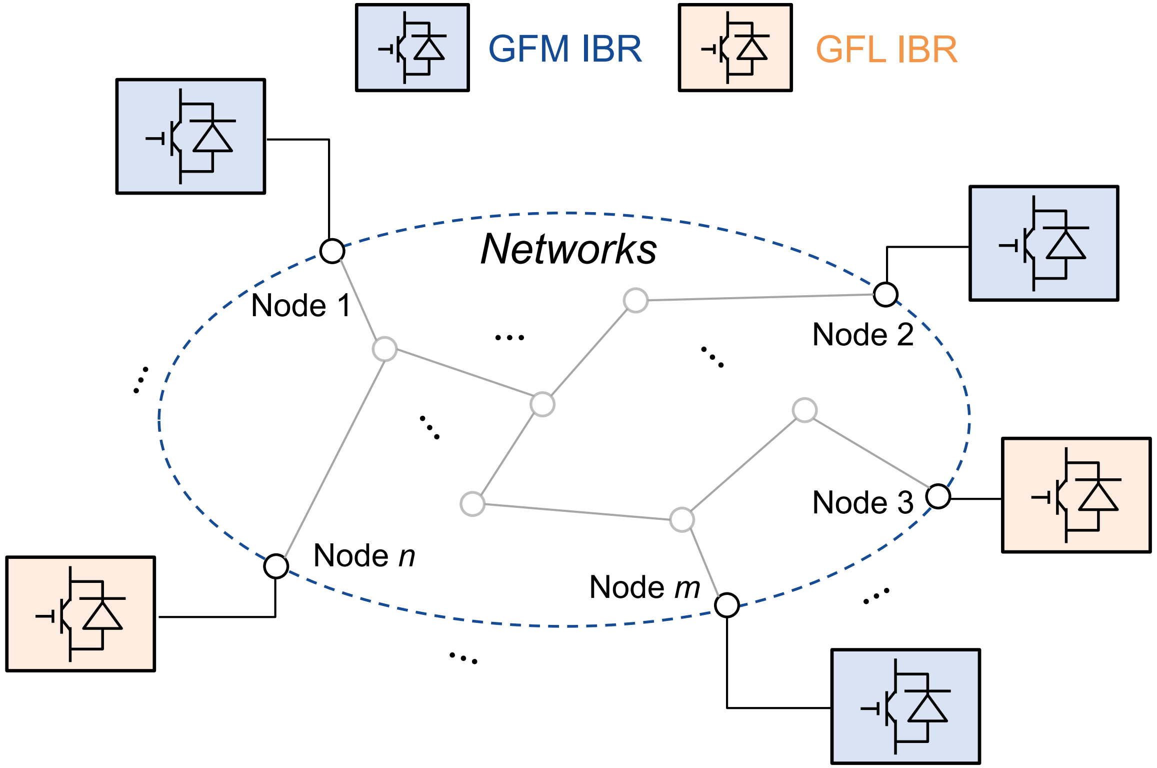
Part II: Applications

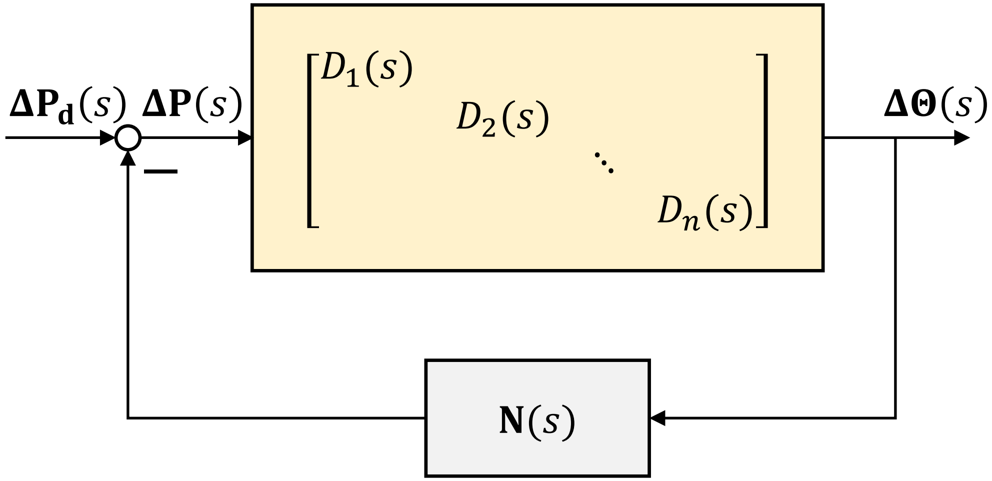
Outline



Mixed GFM-GFL Heterogeneous Power System
$\begin{bmatrix}\Delta \mathbf{P_d}(s) \\ \Delta \mathbf{Q_d}(s)\end{bmatrix}$
-
$\begin{bmatrix}\Delta \mathbf{\Theta}(s) \\ \Delta \mathbf{V}(s)\end{bmatrix}$
Dynamics of IBRs
$\mathbf{N}(s)$
Networks
Part I: Theoretical Analysis
Part II: Applications

Closed-Loop Modeling for Heterogeneous System





Modeling assumptions:

- Considering small-signal frequency dynamics + constant voltages
- Kron-reduced network, retaining GFM/GFL nodes only

Allowing for more general extensions

Preliminaries

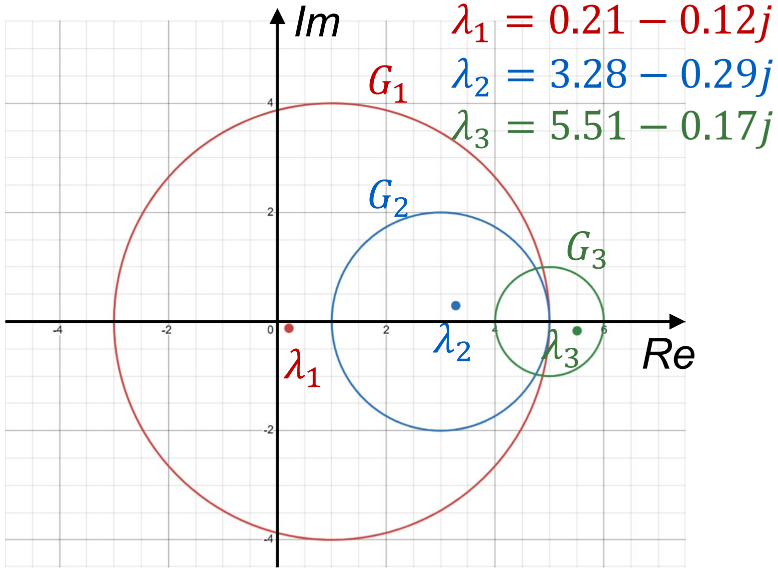
How to estimate the distribution of eigenvalues of a matrix?

$$\mathbf{M} = \begin{bmatrix} 1 & -2 & -2 \\ -1 & 3 & j \\ 0 & -j & 5 \end{bmatrix}$$

Find circles →

$$\begin{cases} G_1 = |\lambda_1 - 1| \leq 4 \\ G_2 = |\lambda_2 - 3| \leq 2 \\ G_3 = |\lambda_3 - 5| \leq 1 \end{cases}$$

Estimate the eigenvalues →



Gershgorin Theorem: For a matrix $\mathbf{M}$, its eigenvalues must lie within the region formed by all the "Gershgorin Circles".

$$G_i = \left\{\lambda_i \in \mathbb{C} \,\middle|\, |\lambda_i - \underline{M_{ii}}| \leq \underline{\sum_{j \neq i} |M_{ij}|}\right\}$$

M_{ii} ↓ diagonal elements

$\sum_{j \neq i} |M_{ij}|$ ↓ sum of moduli of off-diagonal elements

Preliminaries

Gershgorin Theorem: For a matrix $\mathbf{M}$, its eigenvalues must lie within the region formed by all the "Gershgorin Circles".

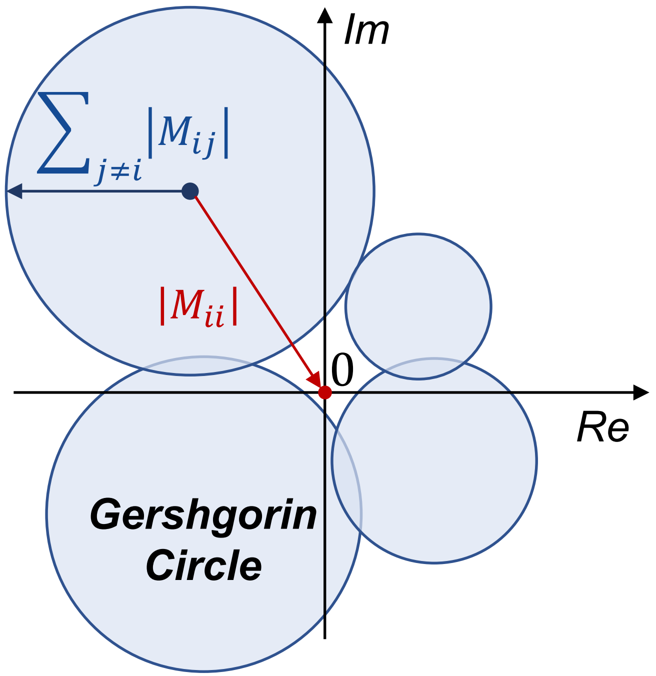
$$G_i = \left\{ \lambda_i \in \mathbb{C} \middle| |\lambda_i - M_{ii}| \le \sum_{j \neq i} |M_{ij}| \right\}$$

$$|M_{ii}| > \sum_{i \neq j} |M_{ij}|, i = 1,2,\ldots,n$$

$$\Rightarrow \lambda_i \neq 0, i = 1,2,\ldots,n$$

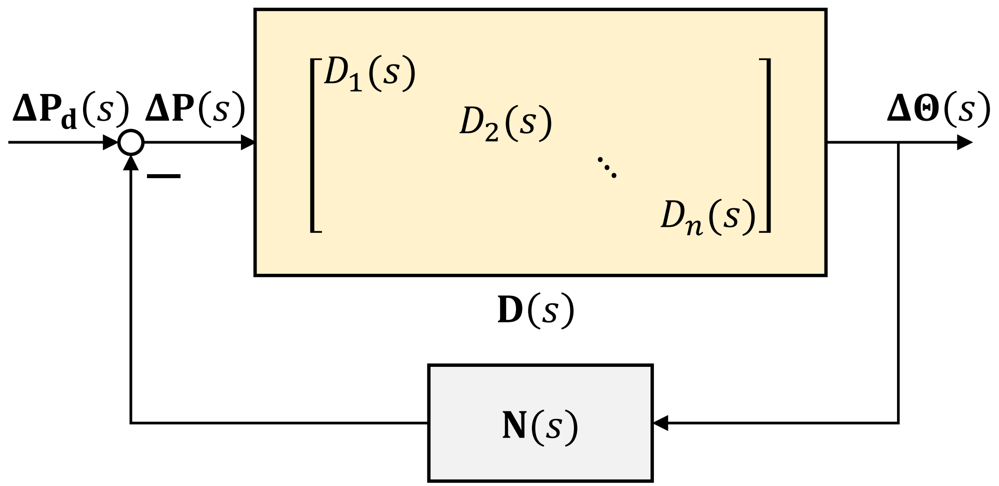
Sufficient Condition

Non-singular: $\det(\mathbf{M}) \neq 0$



Levy-Desplanques Theorem: If $\mathbf{M}$ satisfies the strictly diagonally dominant matrix (DDM) condition, then $\mathbf{M}$ is non-singular.

Proposed Local Gain Condition (LGC)



- Characteristic equation: $\det(\mathbf{I}+\mathbf{D}(s)\mathbf{N}(s))=0$

$$\Rightarrow \det(\mathbf{D}(s))\det\left((\mathbf{D}(s))^{-1}+\mathbf{N}(s)\right)=0$$

- DDM (diagonally dominant matrix) conditions:

$$\Rightarrow \underbrace{\left|(D_i(s))^{-1}+N_{ii}(s)\right|}_{\text{Decoupled local gain}} > \underbrace{\sum_{j\neq i}\left|N_{ij}(s)\right|}_{\text{Coupling gain}} > 0, \forall s \in \Gamma$$

Decoupled local gain counteracts Coupling gain

$$\Rightarrow \det(\mathbf{I}+\mathbf{D}(s)\mathbf{N}(s)) \neq 0, \forall s \in \Gamma \text{ (e.g., RHP)}$$

***Local Gain Condition (LGC)*:** Assume that node dynamics |D(s)| are non-singular within a prohibited domain Γ (e.g., RHP). Thus, if the LGC condition is satisfied in Γ for each IBR, no closed-loop poles lie within this domain.

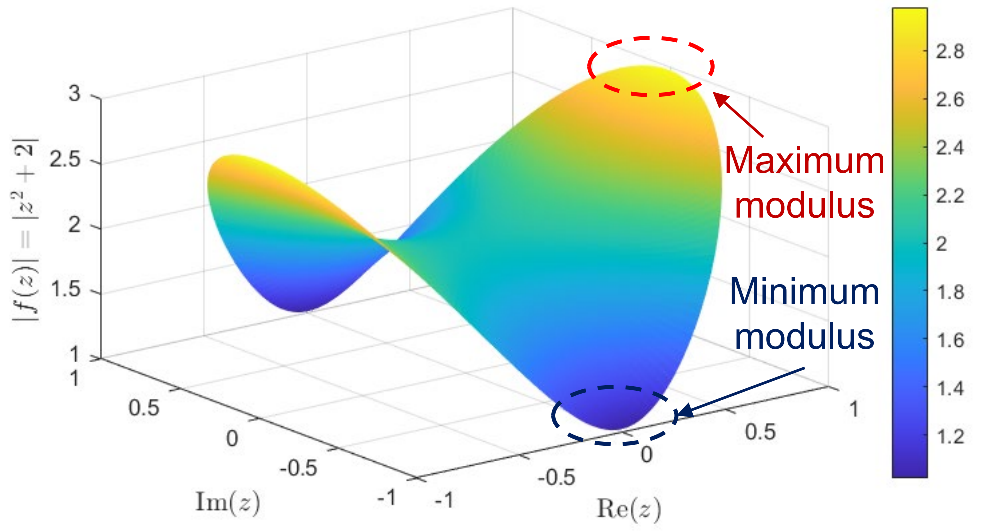
From Domain to Boundary

LGC: $\left|(D_i(s))^{-1} + N_{ii}(s)\right| > \sum_{j\neq i}\left|N_{ij}(s)\right|, \forall s \in \Gamma$

How to solve the LGC-derived complex-function modulus inequality in the prohibited domain Γ?

Maximum Modulus Principle: If a complex function is **analytic** in a bounded closed domain, the **maximum** of the function's modulus is achieved solely on the **boundary**.

Minimum Modulus Principle: If a complex function is **analytic** and **non-zero** in a bounded closed domain, the **minimum** of the function's modulus is also achieved on the **boundary**.

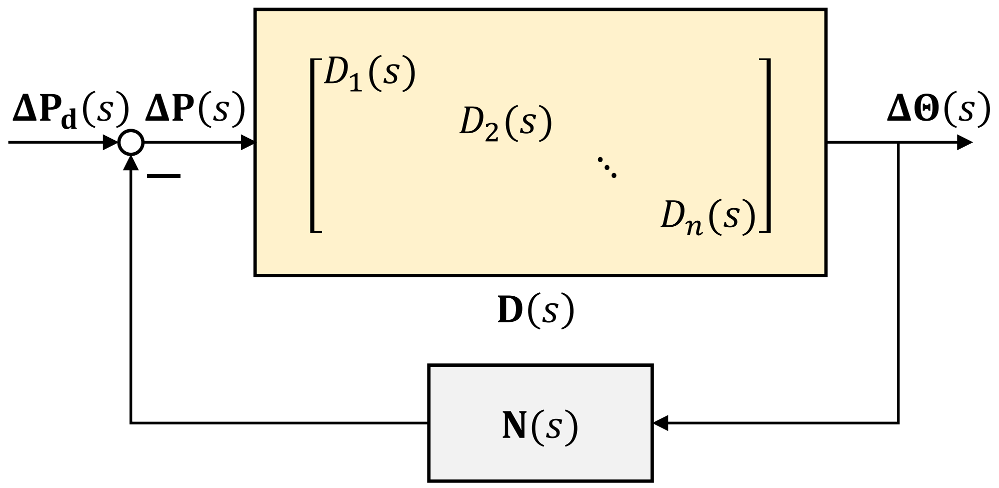


$$\max|f(s)| \Leftrightarrow \min\left|\frac{1}{f(s)}\right|$$

Remark: The infinite boundary can be regarded as a sufficiently large length *R*:

$$\text{RHP}: \{z \in \mathbb{C} | 0 \leq \text{Re}(z) \leq R, |\text{Im}(z)| \leq R, R \to \infty\}$$

Local Gain Boundary Condition (LGBC)



- LGBC:

$$\begin{cases} D_i^{-1}(s) + N_{ii}(s) \neq 0, \forall s \in \Gamma \\ \left|D_i^{-1}(s) + N_{ii}(s)\right| > \sum_{i \neq j} \left|N_{ij}(s)\right|, \forall s \in \mathrm{B} \end{cases}$$

Sufficient Condition

- LGC:

$$\left|(\mathbf{D}_{ii}(s))^{-1} + \mathbf{N}_{ii}(s)\right| > \sum_{i \neq j} \left|\mathbf{N}_{ij}(s)\right|, \forall s \in \Gamma$$

Local Gain Boundary Condition (LGBC): If each diagonal element of ($D^{-1}(s)$+N(s)) is non-zero, then the verification of the LGC only needs to be performed on the boundary of Γ, e.g., $B = j\omega$.

LGBC vs. Small-Gain Condition

Example: Laplacian network N & $\Gamma = \mathrm{RHP}$

Small-gain condition [2] and LGBC require **$D^{-1}(s)$ to fall outside their respective "circles".**

- Small-gain circle:

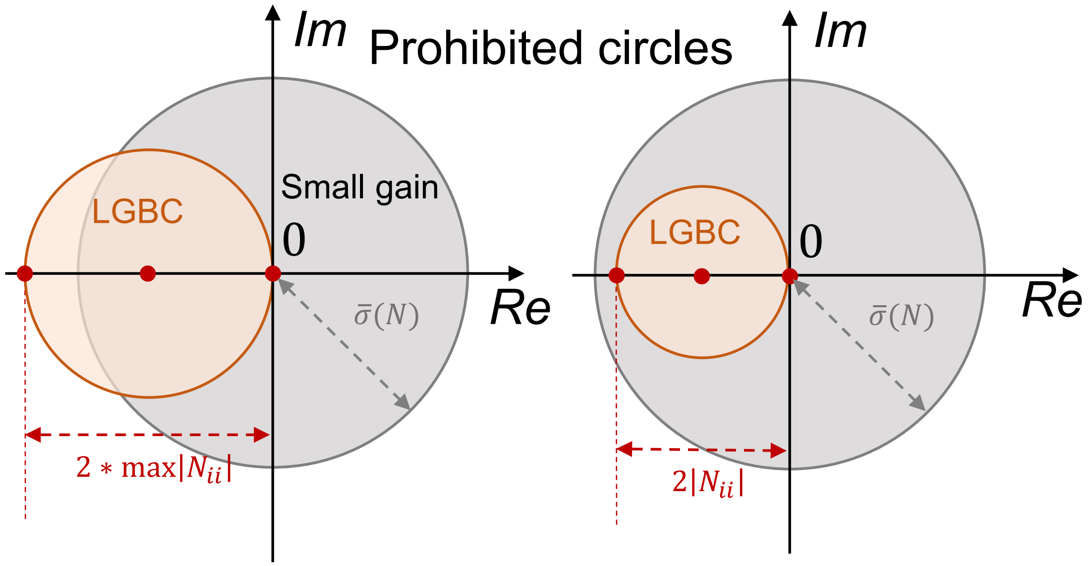
$$\left|D_i^{-1}(s)\right| > \text{max singular value } \bar{\sigma}(\mathbf{N}), \forall s = j\omega$$

- LGBC circles:

$$\left|D_i^{-1}(s) + N_{ii}\right| > \sum_{j\neq i} \left|N_{ij}\right|, \forall s = j\omega$$

- Radius comparison:

$$\bar{\sigma}(N) = \max\big(\lambda(N)\big) \geq \max(N_{ii})$$



Remark:

- The small-gain circle adopt a "**one-size-fits-all**" design
- LGBC circles features a "**locally tailored**" requirement, customized individually for each device
- The largest LGBC circle is **smaller than** the small-gain circle

[2] L. Huang, D. Wang, X. Wang, H. Xin, P. Ju, K. H. Johansson, and F. Dörfler, Gain and phase: Decentralized stability conditions for power electronics-dominated power systems, IEEE Trans. on Power Systems, 39(6): 7240--7256, 2024

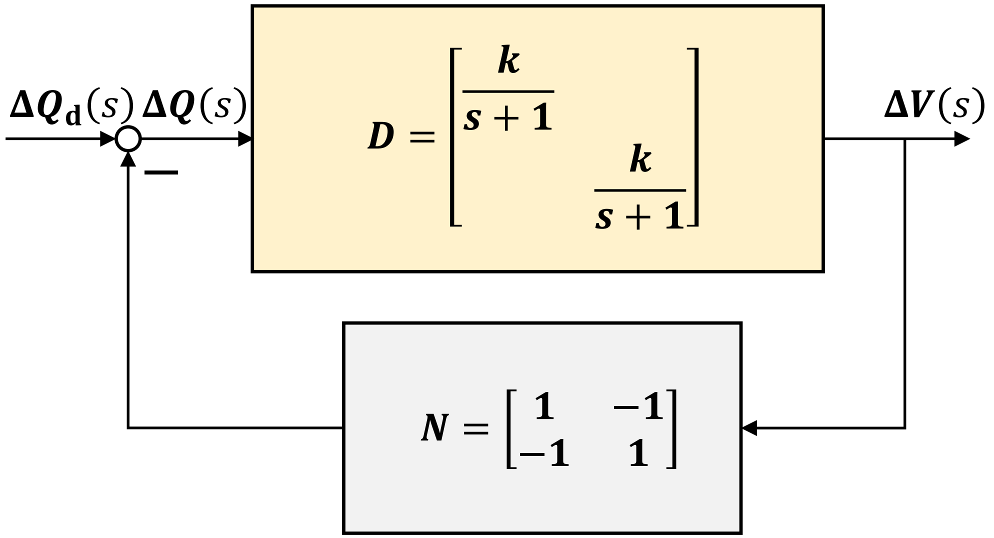
LGBC vs. Small-Gain Condition / Passivity Condition

A toy example: Voltage stability with static Laplacian network N & $\Gamma = \mathrm{RHP}$

- Closed-loop transfer function matrix:

$$G(s) = (I + DN)^{-1} D$$
$$= \frac{k}{(s+2k+1)(s+1)} \begin{bmatrix} s+1+k & k \\ k & s+1+k \end{bmatrix}$$

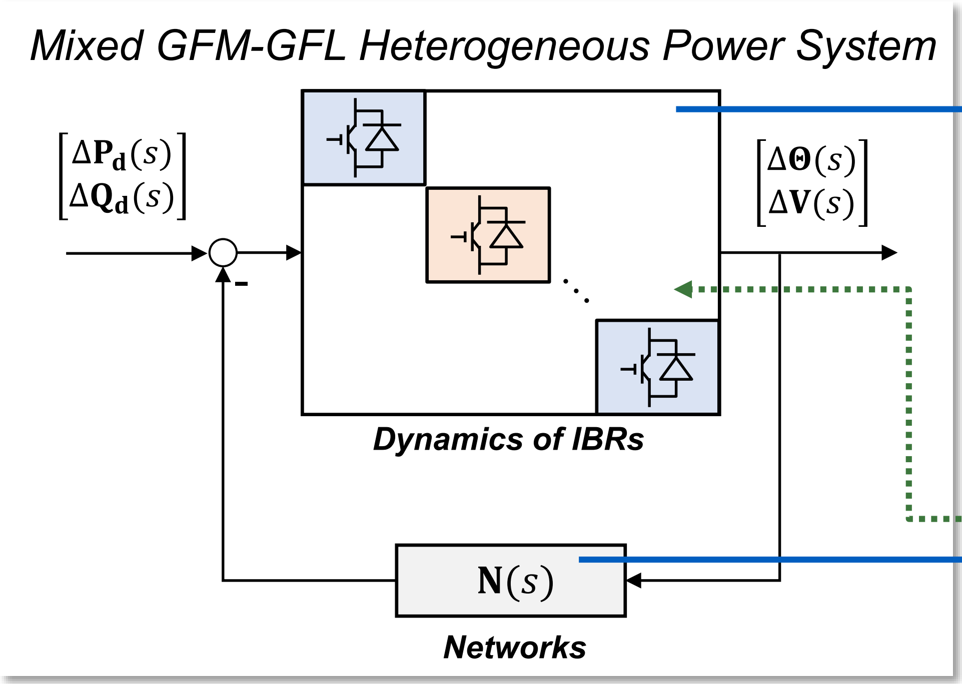
- Exact stability condition: $k > -\frac{1}{2}$
- Passivity-based stability condition: $k > 0$
- Small-gain-based stability condition: $2|k| < 1 \ \rightarrow -\frac{1}{2} < k < \frac{1}{2}$
- LGBC-based stability condition: $\begin{cases} k > -1 \\ \left|\frac{j\omega+1}{k} + 1\right| > 1 \end{cases} \rightarrow k > -\frac{1}{2}$



LGBC:

$$\begin{cases} D_i^{-1}(s) + N_{ii}(s) \neq 0, \forall s \in \Gamma \\ \left|D_i^{-1}(s) + N_{ii}(s)\right| > \sum_{i \neq j} \left|N_{ij}(s)\right|, \forall s \in \mathrm{B} \end{cases}$$

Outline



Part I: Theoretical Analysis

Part II: Applications

Definition of the Prohibited Domain

- Stability-defined prohibited domain (RHP):
  Parameterize the LGBC on the imaginary $s = j\omega$ analytically

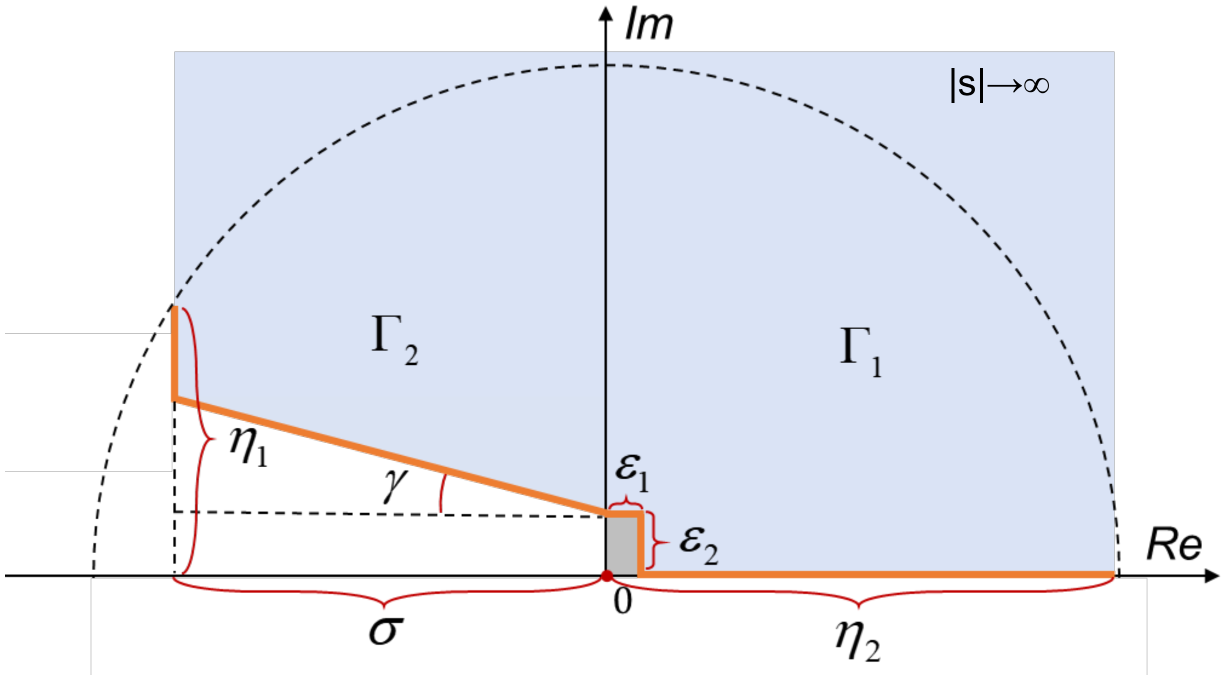
- Custom-defined prohibited domain:

$$\Gamma = \Gamma_1 \cup \Gamma_2$$

$$\begin{cases} \Gamma_1 = \{s \mid \mathrm{Re}(s) \geq 0, \mathrm{Im}(s) \geq 0, s \neq 0\} \\ \Gamma_2 = \left\{s \mid -\sigma \leq \mathrm{Re}(s) < 0, \dfrac{\mathrm{Im}(s)}{-\mathrm{Re}(s)} \geq \tan\gamma \right\} \end{cases}$$

Verify the LGBC on its finite boundary:

$$B^f = \begin{cases} \mathrm{Re}(s) = -\sigma, \sigma\tan\gamma + \varepsilon_2 \leq \mathrm{Im}(s) \leq \eta_1 \\ -\sigma \leq \mathrm{Re}(s) \leq 0, \mathrm{Im}(s) = \varepsilon_2 - \mathrm{Re}(s)\tan\gamma \\ 0 \leq \mathrm{Re}(s) \leq \varepsilon_1, \mathrm{Im}(s) = \varepsilon_2 \\ \mathrm{Re}(s) = \varepsilon_1, 0 \leq \mathrm{Im}(s) \leq \varepsilon_2 \\ \varepsilon_1 \leq \mathrm{Re}(s) \leq \eta_2, \mathrm{Im}(s) = 0 \end{cases}$$



Parameter Feasible Regions Determination

Algorithm 1 Parameter feasible region determination.

Input: Boundary $\mathcal{B}^f$, device matrix $\mathbf{D}(s)$ and network $\mathbf{N}(s)$.

Output: Parameter feasible regions $\mathcal{F} = \{\mathcal{F}_1, \ldots, \mathcal{F}_n\}$.

```
Calculate parameter ranges {Ξ_i}_{i=1}^n that ensure |D_i^{-1}(s, Ξ_i) + N_ii(s)| ≠ 0, Re(s) > −σ.
Discretize boundary B^f into point set S.
for device i ← 1 to n do
    F_i ← ∅ and discretize Ξ_i into candidate set Ψ_i.
    for each parameter ψ ∈ Ψ_i. do
        if |D_i^{-1}(s, ψ) + N_ii(s)| > Σ_{j≠i} |N_ij(s)|, ∀s ∈ S. then
            F_i ← F_i ∪ {ψ}.
        end if
    end for
end for
return F
```

Calculate parameter ranges $\{\Xi_i\}_{i=1}^n$ that ensure $\left|D_i^{-1}(s,\Xi_i) + N_{ii}(s)\right| \neq 0, \mathrm{Re}(s) > -\sigma$.

Discretize boundary $\mathcal{B}^f$ into point set $\mathcal{S}$.

for device $i \leftarrow 1$ to n **do**

$\mathcal{F}_i \leftarrow \emptyset$ and discretize Ξ_i into candidate set Ψ_i.

for each parameter $\psi \in \Psi_i$. **do**

if $\left|D_i^{-1}(s,\psi) + N_{ii}(s)\right| > \sum_{j \neq i} |N_{ij}(s)|, \forall s \in \mathcal{S}$. **then**

$\mathcal{F}_i \leftarrow \mathcal{F}_i \cup \{\psi\}$.

end if

end for

end for

return $\mathcal{F}$

Parallel computation by IBRs

High efficiency regardless of scale

Converting Parameter Feasible Region to Constraint

Main idea: Introduce non-negative variables to construct **mixed-integer linear constraints**, which can be directly applied to planning or scheduling problems.

- First, segment the parameter domain.
  For each segment:

  - Abscissa vector: $X = (x_1 \quad x_2 \quad \cdots \quad x_n)$

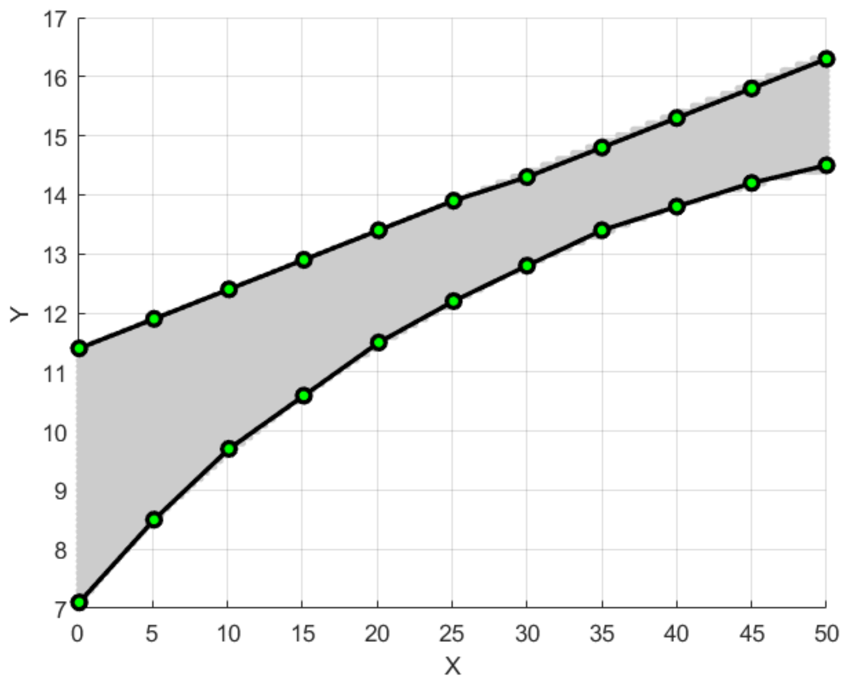
  - Upper bound vector of the ordinate: $Y^U = (y_1^u \quad y_2^u \quad \cdots \quad y_n^u)$

  - Lower bound vector of the ordinate: $Y^L = (y_1^l \quad y_2^l \quad \cdots \quad y_n^l)$

- Then construct the linear constraint:

$$\begin{cases} x = \sum_{i=1}^{n} \lambda_i x_i \\ \sum_{i=1}^{n} \lambda_i y_i^l \le y \le \sum_{i=1}^{n} \lambda_i y_i^u \\ \sum_{i=1}^{n} \lambda_i = 1, \mathrm{SOS}(2) \\ \lambda \ge 0 \end{cases}$$

For parameter optimization application, etc.



SOS(2): At most two adjacent weights are non-zero

Two-IBR Case Study

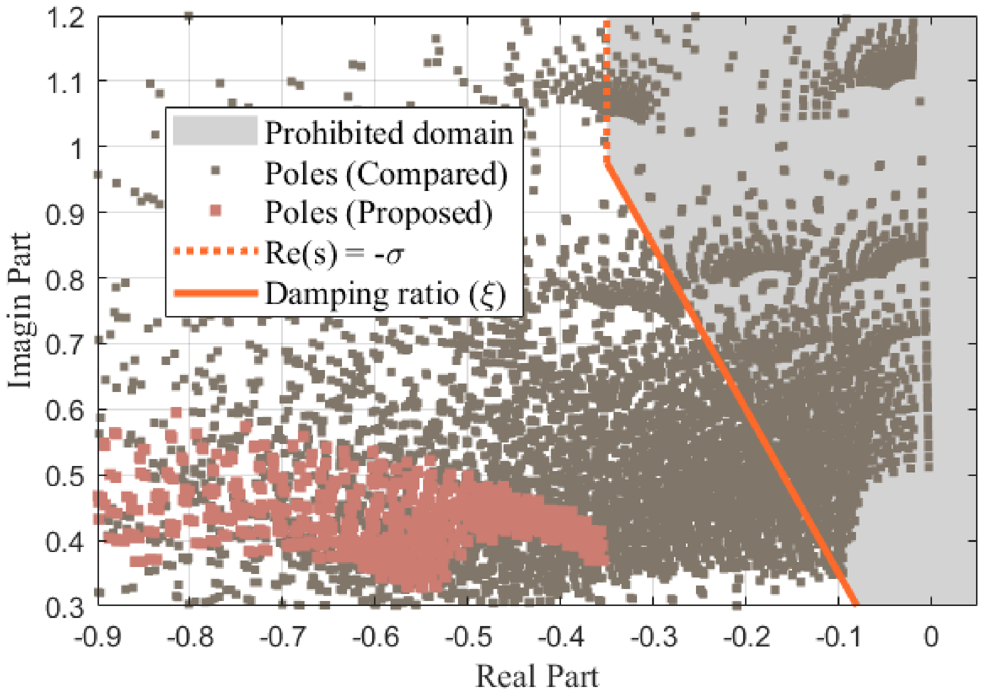
- Primarily focus on low-frequency oscillation (LFO)
- The dynamics of GFM and GFL are defined as:

$$D_i^g(s) = -\frac{1}{s(m_i s + d_i)}$$

$$\left(D_i^f(s)\right)^{-1} = -\frac{s^2 + V_0 K_i^P s + V_0 K_i^I}{s(H_i s + D_i)\left(V_0 K_i^P s + V_0 K_i^I\right)}$$

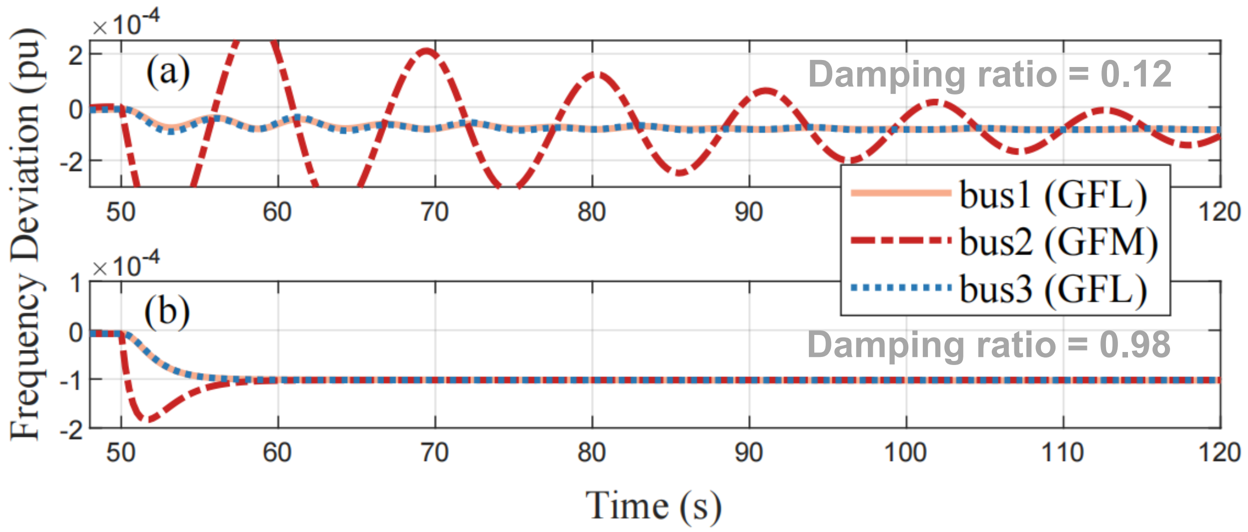
Parameter	Value	Parameter	Value	Parameter	Value
σ	0.35	η_1	10	H_1	0.1~20 (s²/rad)
ξ	0.37	η_2	10	D_1	0.1~20 (s/rad)
K^P_1, K^I_1	4, 40	m	0.1~20 (s²/rad)	H_2	0.1~20 (s²/rad)
K^P_2, K^I_2	2, 20	d	0.1~20 (s/rad)	D_2	0.1~20 (s/rad)

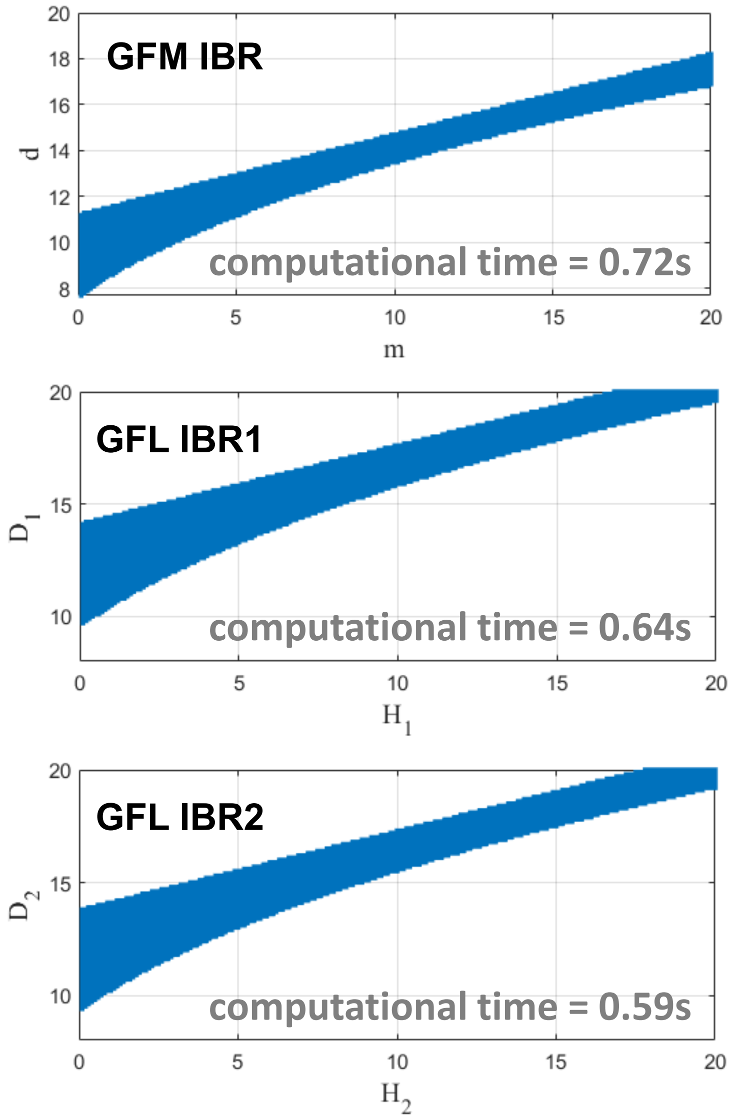
- **No poles lie within the predefined prohibited domain**
- **Instability and weakly damped oscillations are avoided**



Three-IBR Case Study

- A three-IBR test system (one GFM IBR, two GFL IBR) is used to verify the proposed conditions
- **The default parameters lead to weakly-damped poles lying in the prohibited domain (Fig (a))**
- **LGC-derived parameters ensure a required damping ratio (Fig (b))**
- **High efficiency for calculating parameter feasible regions of IBRs**

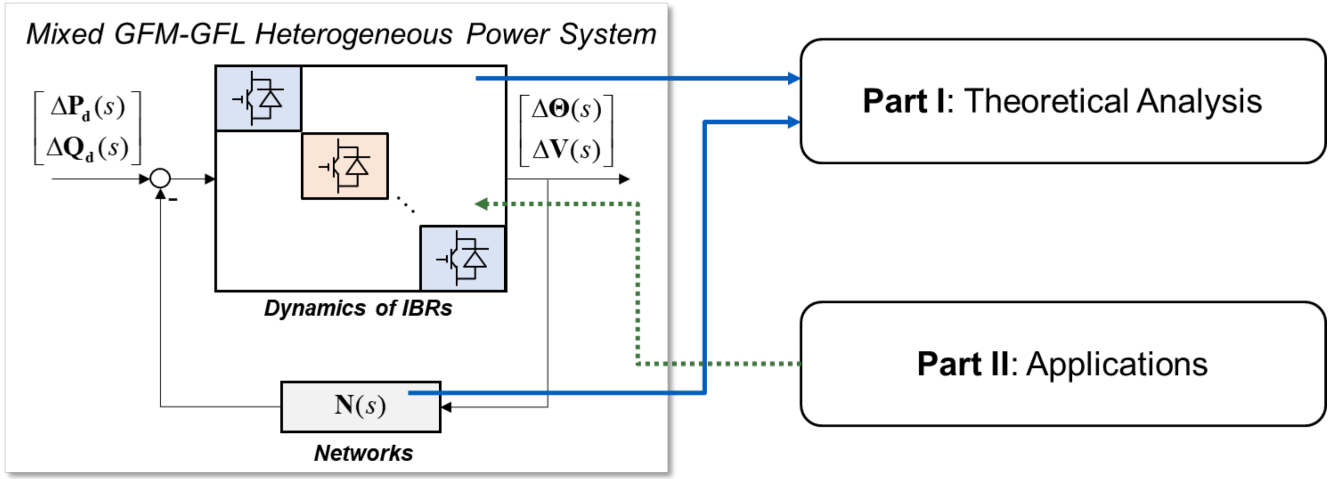




Conclusions and Future Work

- **Part I: Theoretical Analysis**
  - ✓ Local gain conditions (LGC)
  - ✓ Local gain boundary conditions (LGBC)

- **Part II: Applications**
  - ✓ Definition of a prohibited domain
  - ✓ Algorithm for determining the parameter feasible regions
  - ✓ Quantifying as parametric constraints for optimization



Future Work

- Incorporate **voltage dynamics** in the modeling
- Incorporate phase information in the decentralized stability conditions, in addition to norm/magnitude information
- Compare with **other decentralized approaches (Small gains, small phases, SRGs, Davis–Wielandt shells)**
- Optimize frequency and voltage control with parametric stability conditions

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Q & A

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